

# Parallel Programming

Prof. Dr. Uwe Kastens

Winter 2014 / 2015

## Objectives

The participants are taught to understand and to apply

- **fundamental concepts** and **high-level paradigms** of parallel programs,
- **systematic methods** for developing parallel programs,
- **techniques** typical for parallel programming in Java;
- English language in a lecture.

### Exercises:

- The exercises will be tightly integrated with the lectures.
- Small teams will solve given assignments practically supported by a lecturer.
- Homework assignments will be solved by those teams.

## Contents

<b>Week</b>	<b>Topic</b>
1	1. Introduction
2	2. Properties of Parallel Programs
4	3. Monitors in General and in Java
5	4. Systematic Development of Monitors
6	5. Data Parallelism: Barriers
7	6. Data Parallelism: Loop Parallelization
11	7. Asynchronous Message Passing
12	8. Messages in Distributed Systems
14	9. Synchronous message Passing
	10. Conclusion

## Prerequisites

<b>Topic</b>	<b>Course that teaches it</b>
practical experience in programming Java	Grundlagen der Programmierung 1, 2
foundations in parallel programming	Grundlagen der Programmierung 2, Konzepte und Methoden der Systemsoftware (KMS)
process, concurrency, parallelism, interleaved execution	KMS KMS
address spaces, threads, process states	KMS
monitor	KMS
process, concurrency, parallelism, threads,	GP, KMS GP, KMS
synchronization, monitors in Java	GP, KMS
verification of properties of programs	Modellierung

# Organization of the course

## Lecturer

**Prof. Dr. Uwe Kastens:**

Office hours: on appointment by email

**Teaching Assistant:**

- [Peter Pfahler](#)

## Lecture

- V2 Mon 11:15 - 12:45, F1.110

Start date: Oct 13, 2014

## Tutorials

- Grp 1 Mon 09.30 - 11.00 Even Weeks, F2.211 / F1 pool, Start Oct. 27
- Grp 2 Fri 11.00 - 12.30 Odd Weeks, F2.211 / F1 pool, Start Oct. 24

### Schedule

Tutorial	Group 1, Mon 09:30	Group 2, Fri 11:00
1	Oct 27	Oct 24
2	Nov 10	Nov 07
3	Nov 24	Nov 21
4	Dec 08	Dec 05
5	Jan 05	Dec 19
6	Jan 19	Jan 16
7	Feb 02	Jan 30

## Examination

Oral examinations of 20 to 30 min duration. For students of the Computer Science Masters Program the examination is part of a module examination, see [Registering for Examinations](#). In general the examination is held in English. As an alternative, the candidates may choose to give a short presentation in English at the begin of the exam; then the remainder of the exam is held in German. In this case the candidate has to ask via email for a topic of that presentation latest a week before the exam.

# Literature

Course material „**Parallel Programming**“

<http://ag-kastens.upb.de/lehre/material/ppje>

Course material „Grundlagen der Programmierung“ (in German)

Course material „**Software-Entwicklung I + II**“ WS, SS 1998/1999:(in German)

<http://ag-kastens.upb.de/lehre/material/swei>

Course material „**Konzepte und Methoden der Systemsoftware**“ (in German)

Course material „**Modellierung**“ (in German)

<http://ag-kastens.upb.de/lehre/material/model>

Gregory R. Andrews: **Concurrent Programming**, Addison-Wesley, 1991

Gregory R. Andrews: **Foundations of multithreaded, parallel, and distributed programming**, Addison-Wesley, 2000

David Gries: **The Science of Programming**, Springer-Verlag, 1981

Scott Oaks, Henry Wong: **Java Threads**, 2nd ed., O'Reilly, 1999

Jim Farley: **Java Distributed Computing**, O'Reilly, 1998

Doug Lea: **Concurrent Programming in Java**, Addison-Wesley, 2nd Ed., 2000

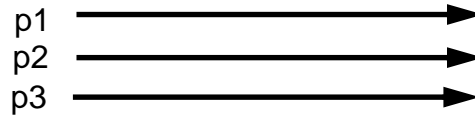
## Fundamental notions (repeated): Parallel processes

### process:

Execution of a sequential part of a program in its storage (address space).  
Variable state: contents of the storage and the position of execution

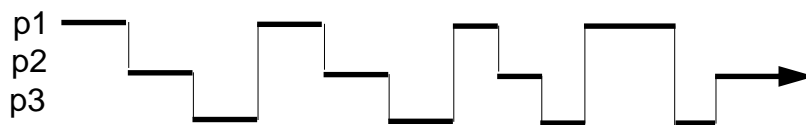
### parallel processes:

several processes, which are executed simultaneously on several processors



### interleaved processes:

several processes, which are executed piecewise alternately on a single processor  
processes are switched by a common process manager or by the processes themselves.

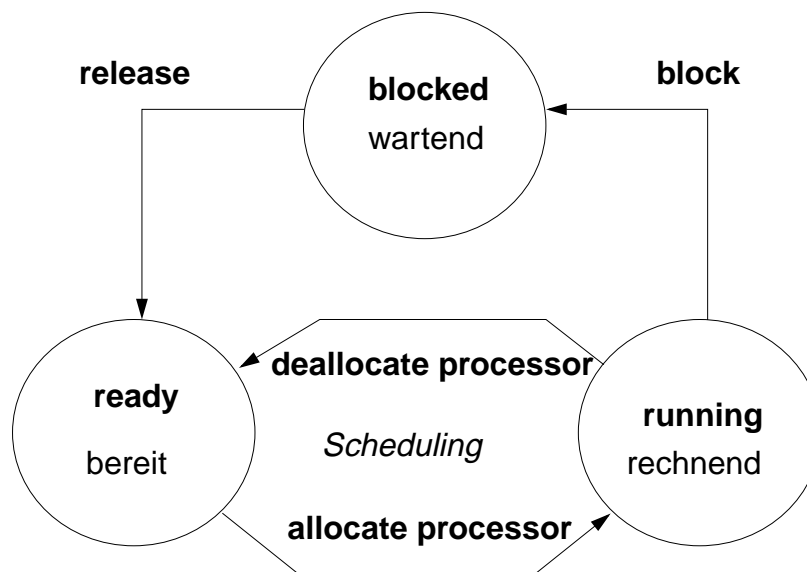


interleaved execution can simulate parallel execution;  
frequent process switching gives the illusion that all process execute steadily.

### concurrent processes:

processes, that can be executed in parallel or interleaved

## Fundamental notions (repeated): States and transitions of processes



see KMS 2-17, 2-18

### Threads (lightweight processes, Leichtgewichtsprozesse):

Processes, that are executed in parallel or interleaved in one common address space;  
process switching is easy and fast.

## Applications of parallel processes

- **Event-based user interfaces:**  
Events are propagated by a specific process of the system.  
Time consuming computations should be implemented by concurrent processes,  
to avoid blocking of the user interface.
- **Simulation** of real processes:  
e. g. production in a factory
- **Animation:**  
visualization of processes, algorithms; games
- **Control** of machines in **Real-Time:**  
processes in the computer control external facilities,  
e. g. factory robots, airplane control
- **Speed-up of execution** by parallel computation:  
several processes cooperate on a common task,  
e. g. parallel sorting of huge sets of data

The application classes follow **different objectives**.

## Create threads in Java - technique: implement Runnable

### Processes, threads in Java:

concurrently executed in the **common address space** of the program (or applet),  
**objects** of class `Thread` with certain properties

### Technique 1: A user's class implements the interface `Runnable`:

```
class MyTask implements Runnable
{
    ...
    public void run ()           The interface requires to implement the method run
    {...}                       - the program part to be executed as a process.
    public MyTask(...) {...}    The constructor method.
}
```

The process is created as an **object of the predefined class `Thread`**:

```
Thread aTask = new Thread (new MyTask (...));
```

The following call starts the process:

```
aTask.start();
```

The new process starts executing in parallel with the initiating one.

This technique (implement the interface `Runnable`) should be used if

- the **new process need not be influenced** any further;  
i. e. it performs its task (method `run`) and then terminates, or
- the **user's class is to be defined as a subclass** of a class different from `Thread`

## Create threads in Java - technique: subclass of Thread

### Technique 2:

The user's class is defined as a **subclass of the predefined class Thread**:

```
class DigiClock extends Thread
{ ...
  public void run ()                Overrides the Thread method run.
  {...}                             The program part to be executed as a process.
  DigiClock (...) {...}            The constructor method.
}
```

The process is created as an **object of the user's class** (it is a **Thread** object as well):

```
Thread clock = new DigiClock (...);
```

The following call starts the process:

```
clock.start();    The new process starts executing in parallel with the initiating one.
```

This technique (subclass of **Thread**) should be used if the new process **needs to be further influenced**; hence, **further methods** of the user's class are to be defined and called from outside the class, e. g. to interrupt the process or to terminate it. The class can not have another superclass!

## Important methods of the class Thread

```
public void run ();
```

is to be overridden with a method that contains the code to be executed as a process

```
public void start ();
```

starts the execution of the process

```
public void suspend ();
(deprecated, deadlock-prone),
suspends the indicated process temporarily: e. g. clock.suspend();
```

```
public void resume ();
(deprecated), resumes the indicated process: clock.resume();
```

```
public void join () throws InterruptedException;
```

the calling process waits until the indicated process has terminated

```
try { auftrag.join(); } catch (Exception e){}
```

```
public static void sleep (long millisec) throws InterruptedException;
```

the calling process waits at least for the given time span (in milliseconds), e. g.

```
try { Thread.sleep (1000); } catch (Exception e){}
```

```
public final void stop () throws SecurityException;
not to be used! May terminate the process in an inconsistent state.
```

## Example: Digital clock as a process in an applet (1)

The process displays the **current date and time** every second as a formatted text.

Applet

Tue Mar 30 18:18:47 CEST 1999

Applet started.

```
class DigiClock extends Thread
{
  public void run ()
  {
    while (running)
    {
      line.setText(new Date().toString());
      try { sleep (1000); } catch (Exception ex) {}
    }
  }
}

```

iterate until it is terminated from the outside  
write the date  
pause

Method, that terminates the process from the outside:

```
public void stopIt () { running = false; }
private volatile boolean running = true;
public DigiClock (Label t) {line = t;}
private Label line;
}

```

state variable  
label to be used for the text

Technique **process as a subclass of Thread**, because it

- **is to be terminated** by a call of `stopIt`,
- **is to be interrupted** by calls of further `Thread` methods,
- other **super classes are not needed**.

## Example: Digital clock as a process in an applet (2)

The process is created in the `init` method of the subclass of `Applet`:

```
public class DigiApp extends Applet
{
  public void init ()
  {
    Label clockText = new Label ("-----");
    add (clockText);

    clock = new DigiClock (clockText);
    clock.start();

    public void start () { /* see below */ }
    public void stop () { /* see below */ }
    public void destroy () { clock.stopIt(); }

    private DigiClock clock;
  }
}

```

create process  
start process  
resume process  
suspend process  
terminate process

Processes, which are started in an applet

- may be suspended, while the applet is invisible (`stop`, `start`);  
better use synchronization or control variables instead of `suspend`, `resume`
- are to be terminated (`stopIt`), when the applet is deallocated (`destroy`).

Otherwise they bind resources, although they are not visible.

## 2. Properties of Parallel Programs

### Goals:

- **formal reasoning** about parallel programs
- **proof properties** of parallel programs
- **develop** parallel programs such that certain **properties can be proven**

### Example A:

```
x := 0; y := 0
co  x := x + 1 //
    y := y + 1
oc
z := x + y
```

Branches of **co-oc** are executed in parallel.

Proof that  $z = 2$  holds at the end.

### Methods:

Hoare Logic, Weakest Precondition, techniques for parallel programs

### Example B:

```
x := 0; y := 0
co  x := y + 1 //
    y := x + 1
oc
z := x + y
```

Show that  $z = 2$  can not be proven.

## Proofs of parallel programs

### Example A:

```
x := 0; y := 0 {x=0 ∧ y=0}
co
{x+1=1}x := x + 1{x=1} //
{y+1=1}y := y + 1{y=1}
oc
{x=1 ∧ y=1} → {x+y=2}
z := x + y {z=2}
```

### Example B<sub>1</sub>:

```
x := 0; y := 0 {x=0 ∧ y=0}
co
{y+1=1}x := y + 1{x=1} //
{x+1=1}y := x + 1{y=1}
oc
{x=1 ∧ y=1} → {x+y=2}
z := x + y {z=2}
```

Check each proof for correctness!

Explain!

### Example B<sub>2</sub>:

```
x := 0; y := 0 {x ≥ 0 ∧ y ≥ 0}
co
{y+1 > 0}x := y + 1{x > 0} //
{x+1 > 0}y := x + 1{y > 0}
oc
{x > 0 ∧ y > 0} → {x+y ≥ 2}
z := x + y {z ≥ 2}
```

Does an **assignment of process p** interfere with an **assertion of process q**?



## Hoare Logic: a brief reminder

Formal calculus for **proving properties of algorithms or programs** [C. A. R. Hoare, 1969]

**Predicates** (assertions) are stated for program positions:

$$\{P\} S_1 \{Q\} S_2 \{R\}$$

A predicate, like  $Q$ , characterizes the **set of states** that any execution of the program can achieve at that position. The predicates are expressions over variables of the program.

Each triple  $\{P\} S \{Q\}$  describes an effect of the execution of  $S$ .  $P$  is called a precondition,  $Q$  a postcondition of  $S$ .

The triple  $\{P\} S \{Q\}$  is correct, if the following holds:

If the execution of  $S$  is begun in a state of  $P$  and **if it terminates**, the the final state is in  $Q$  (partial correctness).

Two special assertions are:

$\{\text{true}\}$  characterizing all states, and  $\{\text{false}\}$  characterizing no state.

Proofs of program properties are constructed using **axioms** and **inference rules** which describe the effects of each kind of statement, and define how proof steps can be correctly combined.

## Axioms and inference rules for sequential constructs

### statement sequence

$$\frac{\begin{array}{l} \{P\} S_1 \{Q\} \\ \{Q\} S_2 \{R\} \end{array}}{\{P\} S_1; S_2 \{R\}} \quad 1$$

### stronger precondition

$$\frac{\begin{array}{l} \{P\} \rightarrow \{R\} \\ \{R\} S \{Q\} \end{array}}{\{P\} S \{Q\}} \quad 3$$

### weaker postcondition

$$\frac{\begin{array}{l} \{P\} S \{R\} \\ \{R\} \rightarrow \{Q\} \end{array}}{\{P\} S \{Q\}} \quad 4$$

### assignment

$$\{P_{[x/e]}\} x := e \{P\} \quad 2$$

$P_{[x/e]}$  means:  $P$  with all free occurrences of  $x$  substituted by  $e$

### multiple alternative (guarded command)

$$\frac{\begin{array}{l} P \wedge \neg(B_1 \vee \dots \vee B_n) \Rightarrow Q \\ \{P \wedge B_i\} S_i \{Q\}, \quad 1 \leq i \leq n \end{array}}{\{P\} \text{ if } B_1 \rightarrow S_1 \ [] \dots \ [] B_n \rightarrow S_n \text{ fi } \{Q\}} \quad 5$$

### selecting iteration

$$\frac{\{INV \wedge B_i\} S_i \{INV\}, \quad 1 \leq i \leq n}{\{INV\} \text{ do } B_1 \rightarrow S_1 \ [] \dots \ [] B_n \rightarrow S_n \text{ od } \{INV \wedge \neg(B_1 \vee \dots \vee B_n)\}} \quad 6$$

### no operation

$$\{P\} \text{ skip } \{P\} \quad 7$$

## Verification: algorithm computes gcd

precondition:  $x, y \in \mathbb{N}$ , i. e.  $x > 0, y > 0$ ; let  $G$  be greatest common divisor of  $x$  and  $y$

postcondition:  $a = G$

algorithm with { assertions over variables }:

{  $G$  is gcd of  $x$  and  $y \wedge x > 0 \wedge y > 0$  }

$a := x; b := y;$

{ INV:  $G$  is gcd of  $a$  and  $b \wedge a > 0 \wedge b > 0$  }

do  $a \neq b \rightarrow$

{ INV  $\wedge a \neq b$  }

if  $a > b \rightarrow$

{  $G$  is gcd of  $a$  and  $b \wedge a > 0 \wedge b > 0 \wedge a > b$  }  $\rightarrow$

{  $G$  is gcd of  $a-b$  and  $b \wedge a-b > 0 \wedge b > 0$  }

$a := a - b$

{ INV }

[ ]  $a \leq b \rightarrow$

{  $G$  is gcd of  $a$  and  $b \wedge a > 0 \wedge b > 0 \wedge b > a$  }  $\rightarrow$

{  $G$  is gcd of  $a$  and  $b-a \wedge a > 0 \wedge b-a > 0$  }

$b := b - a$

{ INV }

fi { INV  $\wedge a \neq b \wedge \neg(a > b \vee a \leq b) \rightarrow$  INV } „there is no 3rd case for the if  $\rightarrow$  INV“

{ INV }

od

{ INV  $\wedge a = b$  }  $\rightarrow$

{  $a = G$  }

the loop terminates:

- $a+b$  decreases monotonic
- $a+b > 0$  is invariant

## Weakest precondition

A similar calculus as Hoare Logic is based on the notion of weakest preconditions [Dijkstra, 1976; Gries 1981]:

Program positions are also annotated by assertions that characterize program states.

The **weakest precondition**  $wp (s, Q) = P$  of a statement  $s$  maps a predicate  $Q$  on a predicate  $P$  ( $wp$  is a **predicate transformer**).

$wp (s, Q) = P$  characterizes **the largest set of states** such that if the execution of  $s$  is begun in any state of  $P$ , then the execution is **guaranteed to terminate** in a state of  $Q$  (**total correctness**).

If  $P \Rightarrow wp (s, Q)$  then  $\{P\} s \{Q\}$  holds in Hoare Logic.

This concept is a more goal oriented proof method compared to Hoare Logic. We need weakest precondition only in the definition of „non-interference“ in proof for parallel programs.

## Examples for weakest preconditions

1.  $P = \text{wp}(\text{statement}, Q)$
2.  $i \leq 0 = \text{wp}(i := i + 1, i \leq 1)$
3.  $\text{true} = \text{wp}(\text{if } x \geq y \text{ then } z := x \text{ else } z := y, z = \max(x, y))$
4.  $(y \geq x) = \text{wp}(\text{if } x \geq y \text{ then } z := x \text{ else } z := y, z = y)$
5.  $\text{false} = \text{wp}(\text{if } x \geq y \text{ then } z := x \text{ else } z := y, z = y - 1)$
6.  $(x = y + 1) = \text{wp}(\text{if } x \geq y \text{ then } z := x \text{ else } z := y, z = y + 1)$
7.  $\text{wp}(S, \text{true}) =$  the set of all states such that the execution of  $S$  begun in one of them is guaranteed to terminate

## Interleaving - used as an abstract execution model

Processes that are not blocked may be switched at **arbitrary points** in time.

A **scheduling strategy** reduces that freedom of the scheduler.

An example shows how different results are exhibited by switching processes differently. Two processes operate on a common variable **account**:

```

account = 50;
      a           b           c
-----
Process1: t1 = account; t1 = t1 + 10; account = t1;
Process2: t2 = account; t2 = t2 - 5; account = t2;
      d           e           f
-----

```

Assume that the assignments  $a - f$  are atomic. Try any interleaved execution order of the two processes on a single processor. Check what the value of **account** is in each case.

Assume the sequences of statements  $\langle a, b \rangle$  and  $\langle d, e \rangle$  (or  $\langle b, c \rangle$  and  $\langle e, f \rangle$ ) are atomic and check the results of any interleaved execution order.

We get the **same variety of results**, because there are **no global variables** in  $b$  or  $e$ . The coarser execution model is sufficient.

## Atomic actions

**Atomic action:** A sequence of (one or more) operations, the internal states of which can not be observed because it has one of the following properties:

- it is a **non-interruptable machine instruction**,
- it has the **AMO** property, or
- **Synchronization** prohibits, that the action is interleaved with those of other processes, i. e. explicitly atomic.

### At-most-once property (AMO):

The construct has **at most one** point where an other process can interact:

- **Expression E:**  
E has at most one variable v, that is written by a different process, and v occurs only once in E.
- **Assignment x := E:**  
E is AMO and x is not read by a different process, or x may be read by a different process, but E does not contain any global variable.
- **Statement sequence S:**  
one statement in S is AMO and all other statements in S do not have any global variable.

## Atomic by AMO

Interleaving analysis is **simpler**, if **atomic decomposition is coarser**.

Check AMO property for nested constructs. Consider the most enclosing one to be atomic.

**Examples:** assume  $x = 0; y = 0; z = 0;$  to be global

atomic AMO constructs  $\langle \dots \rangle$ :

$\langle t = \langle \langle x \rangle + \langle 1 \rangle \rangle; \rangle \langle x = \langle 1 \rangle \rangle;$

**interleaving actions of two processes:**

(1) 
$$\begin{array}{l} \text{p1:} \quad \langle t = 0; t = t + 1; \rangle \\ \text{p2:} \quad \langle s = 0; s = s + 1; \rangle \end{array}$$

*a*  
*b*

(2) 
$$\begin{array}{l} \text{p1:} \quad \langle x = 2; \rangle \\ \text{p2:} \quad \langle t = x + 1; \rangle \end{array}$$

*a*  
*b*

(3) 
$$\begin{array}{l} \text{p1:} \quad x = \langle y + 1 \rangle; \\ \text{p2:} \quad y = \langle x + 1 \rangle; \end{array}$$

*b*   *a*  
*d*   *c*

(4) 
$$\begin{array}{l} \text{p1:} \quad x = \langle y \rangle + \langle z \rangle; \\ \text{p2:} \quad \langle y = 1; \rangle \langle z = 2; \rangle; \end{array}$$

*c*   *a*   *b*  
*d*   *e*

## Interference between processes

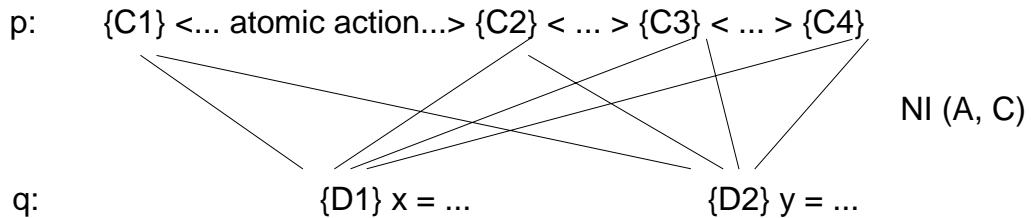
**Critical assertions** characterize **observable states** of a process p:

Let  $\{P\} S \{Q\}$  be the statement sequence of process p with its pre- and postcondition. Then Q is critical.

Let T be a statement in S that is not part of an atomic statement and R its postcondition; then  $C = wp(T, R)$  is critical.

For every critical assertion of the proof of p, it has to be proven that

**non-interference NI (A, C)** holds for each **assignment A** of every other process q:



**non-interference NI (A, C)** holds between

**assignment A:**  $\{D\} x = e$  in q having precondition D in a proof of q and **assertion C** on p, if the following can be proven in programming logic:

$$\{C \wedge D\} A \{C\}$$

i. e. **the execution of A does not interfere with C (can not change C)**, provided that the precondition D allows to execute A in a state where C holds.

## Example: Interference between an assertion and an assignment

Consider processes p and q with **assertions at observable states**.

Consider a single critical **assertion C** in p and a single **assignment A** in q:

p: ... <...>  $\{C\}$  <...> ...

q: ... <...>  $\{d+1 > 0\}$  a = d + 1;  $\{Q\}$  <...> ...  
A

Does A interfere with C? Depends on C:

1. C:  $a == 1$   
 $\{a == 1 \wedge d + 1 > 0\} a = d + 1 \{a == 1\}$  is not provable  $\Rightarrow$  interference  
 $\underset{C}{\quad}$   $\underset{C}{\quad}$
2. C:  $a > 0$   
 $\{a > 0 \wedge d + 1 > 0\} a = d + 1 \{a > 0\}$  is provable  $\Rightarrow$  non-interference
3. C:  $a == 1 \wedge d < 0$   
 $\{a == 1 \wedge d < 0 \wedge d + 1 > 0\} a = d + 1 \{a == 1 \wedge d < 0\}$  is provable  $\Rightarrow$  non-interference  
\_\_\_\_\_  $\underset{f}{\quad}$

## Non-interference checks

```

x := 0; y := 0;
{ x = 0 ∧ y = 0 }
co {x+1 = 1} x := x+1 {x=1} //
   {y+1 = 1} y:= y+1 {y=1}
oc
{ x = 1 ∧ y = 1 } => {x+y = 2}
z := x+y
{z = 2}
    
```

NI(a, c) holds for all 4 cases, e.g.

$\{x+1 = 1 \wedge y+1 = 1\} y:= y+1 \{x+1 = 1 \wedge y = 1\} \Rightarrow \{x+1 = 1\}$

```

x := 0; y := 0;
{ x = 0 ∧ y = 0 }
co {y+1 = 1} x := y+1 {x=1} //
   {x+1 = 1} y:= x+1 {y=1}
oc
{ x = 1 ∧ y = 1 } => {x+y = 2}
z := x+y
{z = 2}
    
```

NI(y:= x+1, y+1 = 1) does not hold:

$\{y+1 = 1 \wedge x+1 = 1\} y:= x+1 \{y+1 = 1\}$   
is not correct

is not correct

## Two inference rules for concurrent execution

The statement for **condition synchronization**

`<await B -> S>`

causes the executing process to be blocked until the condition **B** is true; then **s** is executed. The whole statement is executed as an atomic action; hence **B** holds at the begin of **s**.

$$\frac{\{P \wedge B\} S \{Q\}}{\{P\} \langle \text{await } B \rightarrow S \rangle \{Q\}}$$

The statement for **concurrent processes**

`co S1 // ... // Sn oc`

executes the statements **s<sub>i</sub>** concurrently. It terminates when all **s<sub>i</sub>** have terminated.

**Non-Interference is to be proven.**

$\{P_i\} S_i \{Q_i\}, 1 \leq i \leq n$ , are **interference-free theorems**

$\{P_1 \wedge \dots \wedge P_n\} \text{co } S_1 // \dots // S_n \text{ oc } \{Q_1 \wedge \dots \wedge Q_n\}$

## Avoiding interference

### 1. disjoint variables:

Two concurrent processes  $p$  and  $q$  are interference-free if the set of variables  $p$  writes to is disjoint from the set of variables  $q$  reads from and vice versa.

### 2. weakened assertions:

The assertions in the proofs of concurrent processes can in some cases be made interference-free by weakening them.

### 3. atomic action:

A non-interference-free assertion  $C$  can be hidden in an atomic action.

$p:: \dots x := e \dots$

$p:: \dots x := e \dots$

$q:: \dots s1 \{C\} s2 \dots$

$q:: \dots \langle s1 \{C\} s2 \rangle \dots$

### 4. condition synchronization:

A synchronization condition can make an interfering assignment interference-free.

*S2 can not be executed in this state or C holds after  $x:=e$*

$p:: \dots x := e \dots$

$p:: \dots \langle \text{await not } C \text{ or } B \rightarrow x:=e \rangle \dots$   
with  $B = wp(x:=e, C)$

$q:: \dots s1 \{C\} s2 \dots$

$q:: \dots s1 \{C\} s2 \dots$

## 3. Monitors in general and in Java Communication and synchronization of parallel processes

**Communication** between parallel processes: exchange of data by

- using a common, global variable, only in a programming model with **common storage**
- **messages** in programming model **distributed** or **common storage**  
**synchronous** messages: sender waits for the receiver (languages: CSP, Occam, Ada, SR)  
**asynchronous** messages: sender does not wait for the receiver (languages: SR)

**Synchronization** of parallel processes:

- **mutual exclusion (gegenseitiger Ausschluss):**  
certain statement sequences (critical regions) may not be executed by several processes at the same time
- **condition synchronization (Bedingungssynchronisation):**  
a process waits until a certain condition is satisfied by a different process

**Language constructs for synchronization:**

Semaphore, monitor, condition variable (programming model with common storage)  
messages (see above)

**Deadlock (Verklemmung):**

Some processes are waiting cyclically for each other, and are thus blocked forever

## Monitor - general concept

**Monitor:** high level synchronization concept introduced in [C.A.R. Hoare 1974, P. Brinch Hansen 1975]

### Definition:

- A monitor is a **program module** for concurrent programming with **common storage**; it encapsulates data with its operations.
- A monitor has **entry procedures** (which operate on its data); they are **called by processes**; the monitor is **passive**.
- The monitor guarantees **mutual exclusion for calls of entry procedures**:  
at most one process executes an entry procedure at any time.
- **Condition variables** are defined in the monitor and are used within entry procedures for **condition synchronization**.

## Condition variables

A **condition variable** *c* is defined to have 2 operations to operate on it. They are executed by processes when executing a call of an entry procedure.

- **wait (c)** The executing process **leaves the monitor** and waits in a set associated to *c*, until it is released by a subsequent call `signal(c)`; then the process accesses the monitor again and continues.
- **signal (c):** The executing process releases **one arbitrary process** that waits for *c*.

Which of the two processes immediately continues its execution in the monitor depends on the variant of the signal semantics (see PPJ-22).

### **signal-and-continue:**

The signal executing process continues its execution in the monitor.

A call `signal (c)` has **no effect, if no process is waiting** for *c*.

Condition synchronization usually has the form

```
if not B then wait (c);    or    while not B do wait (c);
```

The **condition variable c** is used to synchronize on the **condition B**.

**Note** the difference between condition variables and semaphores:

Semaphores are counters. The effect of a call `V(s)` on a semaphore is not lost if no process is waiting on *s*.



## Example: bounded buffer

### monitor Buffer

```
buf: Queue (k);
notFull, notEmpty: Condition;      2 condition variables: state of the buffer
```

#### entry put (d: Data)

```
do length(buf) = k -> wait (notFull); od;
enqueue (buf, d);
signal (notEmpty);
end;
```

#### entry get (var d: Data)

```
do length (buf) = 0 -> wait (notEmpty); od;
d := front (buf); dequeue (buf);
signal (notFull);
end;
```

```
end;
```

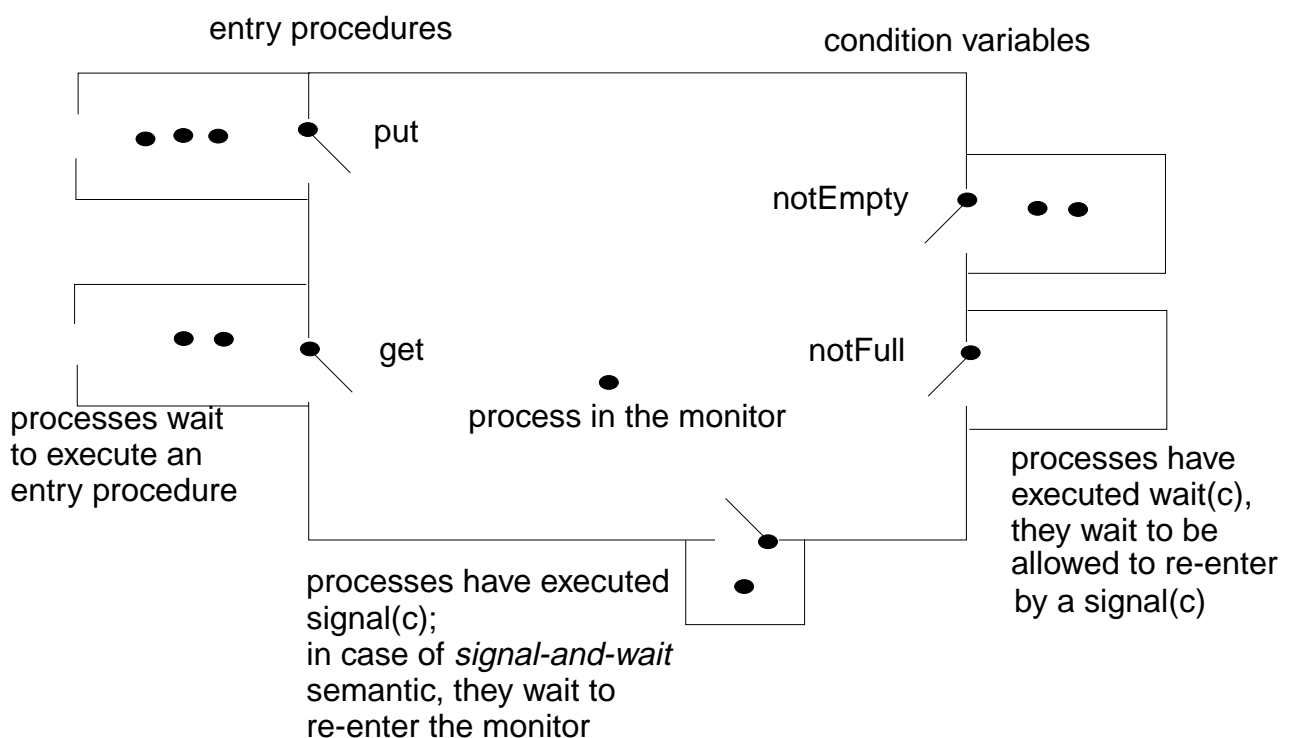
#### process Producer (i: 1..n) d: Data;

```
loop d := produce(); Buffer.put(d); end;
end;
```

#### process Consumer (i: 1..m) d: Data;

```
loop Buffer.get(d); consume(d); end;
end;
```

## Synchronization in a monitor



## Variants of signal-wait semantics

Processes compete for the monitor

- processes that are blocked by executing `wait(c)`,
- process that is in the monitor, may be executing `signal(c)`
- processes that wait to execute an entry procedure

### signal-and-exit semantics:

The process that executes `signal` terminates the entry procedure call and leaves the monitor.

The released process enters the monitor **immediately** - without a state change in between

### signal-and-wait semantics:

The process that executes `signal` leaves the monitor and waits to re-enter the monitor.

The released process enters the monitor **immediately** - without a state change in between

### Variant signal-and-urgent-wait:

The process that has executed `signal` gets a higher priority than processes waiting for entry procedures

### signal-and-continue semantics:

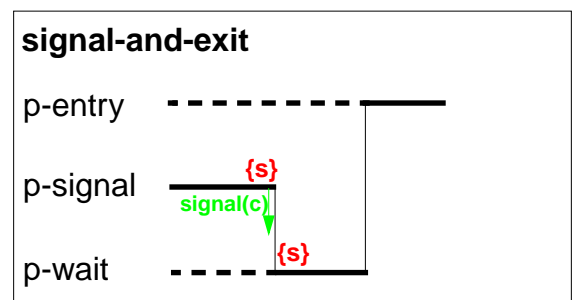
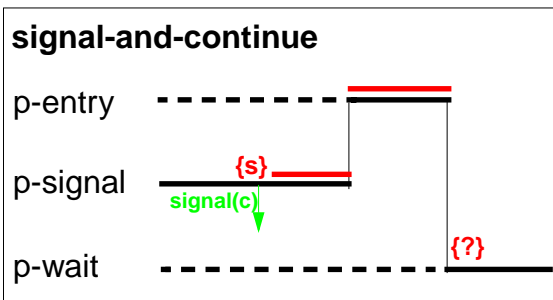
The process that executes `signal` continues execution in the monitor.

The released process has to wait until the monitor is free. The **state** that held at the

`signal` call may be changed meanwhile; the waiting condition has to be checked again:

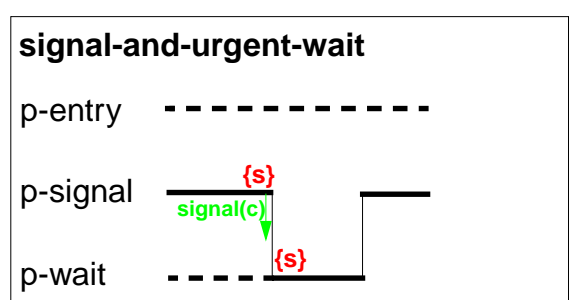
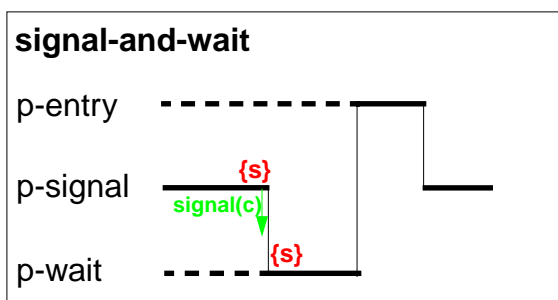
```
do length(buf) = k -> wait(notFull); od;
```

## Variants of signal-wait semantics: examples of execution



3 processes:  
 p-entry waits to enter an entry procedure  
 p-signal executes `signal(c)`  
 p-wait has executed `wait(c)`

**{s}** state when `signal(c)` is executed  
**{s}** may be modified here: —



## Monitors in Java: mutual exclusion

Objects of any class can be used as **monitors**

### Entry procedures:

Methods of a class, which implement critical operations on instance variables can be marked **synchronized**:

```
class Buffer
{   synchronized public void put (Data d) {...}
    synchronized public Data get () {...}
    ...
    private Queue buf;
}
```

If several processes **call synchronized methods** for the same object, they are executed under **mutual exclusion**.

They are synchronized by an internal synchronization variable of the object (lock).

Non-**synchronized** methods can be executed at any time concurrently.

There are also **synchronized class methods**: they are called under mutual exclusion with respect to the class.

**synchronized blocks** can be used to specify execution of a critical region with respect to an arbitrary object.

## Monitors in Java: condition synchronization

All processes that are blocked by **wait** are held in a single set;  
**condition variables can not be declared** (there is only an implicit one)

Operations for condition synchronization:

are to be called from inside **synchronized** methods:

- **wait()**        **blocks** the executing process;  
releases the monitor object, and  
waits in the unique set of blocked processes of the object
- **notifyAll()** releases **all** processes that are blocked by **wait** for this object;  
they then compete for the monitor;  
the executing process continues in the monitor  
(signal-and-continue semantics).
- **notify()**     releases **an arbitrary** one of the processes that are blocked by **wait**  
for this object;  
the executing process continues in the monitor  
(signal-and-continue semantics);  
**only usable if all processes wait for the same condition.**

**Always call wait in loops**, because with **signal-and-continue** semantics  
after **notify**, **notifyAll** the **waiting condition may be changed**:

```
while (!Condition) try { wait(); } catch (InterruptedException e) {}
```

## A Monitor class for bounded buffers

```

class Buffer
{ private Queue buf;                // Queue of length n to store the elements
  public Buffer (int n) {buf = new Queue(n); }

  synchronized public void put (Object elem)
  {                                // a producer process tries to store an element
    while (buf.isFull())          // waits while the buffer is full
      try {wait();} catch (InterruptedException e) {}
    buf.enqueue (elem);          // changes the waiting condition of the get method
    notifyAll();                 // every blocked process checks its waiting condition
  }

  synchronized public Object get ()
  {                                // a consumer process tries to take an element
    while (buf.isEmpty())        // waits while the buffer is empty
      try {wait();} catch (InterruptedException e) {}
    Object elem = buf.first();
    buf.dequeue();               // changes the waiting condition of the put method
    notifyAll();                 // every blocked process checks its waiting condition
    return elem;
  }
}

```

## Concurrency Utilities in Java 2

The **Java 2 platform** includes a *package of concurrency utilities*. These are classes which are designed to be used as building blocks in building concurrent classes or applications. ...

...

**Locks** - While locking is built into the Java language via the synchronized keyword, there are a number of **inconvenient limitations to built-in monitor locks**. The `java.util.concurrent.locks` package provides a high-performance lock implementation with **the same memory semantics as synchronization**, but which also supports specifying a timeout when attempting to acquire a lock, **multiple condition variables per lock**, non-lexically scoped locks, and support for interrupting threads which are waiting to acquire a lock.

<http://java.sun.com/j2se/1.5.0/docs/guide/concurrency/index.html>

<http://java.sun.com/j2se/1.5.0/docs/api/java/util/concurrent/locks/Condition.html>

## Concurrency Utilities in Java 2 (example)

```

class BoundedBuffer {
    final Lock lock = new ReentrantLock();           explicit lock
    final Condition notFull = lock.newCondition();  condition variables
    final Condition notEmpty = lock.newCondition();

    final Object[] items = new Object[100];
    int putptr, takeptr, count;

    public void put (Object x) throws InterruptedException {
        lock.lock();                               explicit mutual exclusion
        try { while (count == items.length) notFull.await();    specific wait
            items[putptr] = x;
            if (++putptr == items.length) putptr = 0;
            ++count;
            notEmpty.signal();                     specific signal
        } finally { lock.unlock();}                explicit mutual exclusion
    }

    public Object get () throws InterruptedException {
        lock.lock();                               explicit mutual exclusion
        try { while (count == 0) notEmpty.await();              specific wait
            Object x = items[takeptr];
            if (++takeptr == items.length) takeptr = 0;
            --count;
            notFull.signal();                         specific signal
        } finally { lock.unlock();}                explicit mutual exclusion
    }
}

```

### 3. Systematic Development of monitors Monitor invariant

A **monitor invariant (MI)** specifies **acceptable states of a monitor**

**MI has to be true whenever a process may leave or (re-)enter the monitor:**

- after the **initialization**,
- at the **beginning** and at the **end of each entry procedure**,
- before and after each call of **wait**,
- before and after each call of **signal** with **signal-and-wait** semantics (\*),
- before each call of **signal** with **signal-and-exit** semantics (\*).

Example of a monitor invariant for the bounded buffer:

$$\text{MI: } 0 \leq \text{buf.length}() \leq n$$

The **monitor invariant has to be proven** for the program positions

after the initialization, at the end of entry procedures, before calls of wait (and signal if (\*)).

One can **assume that the monitor invariant holds** at the other positions

at the beginning of entry procedures, after calls of wait (and signal if (\*)).

## Design steps using monitor invariant

1. Define the **monitor state**, and design the **entry procedures without synchronization**  
e. g. bounded buffer: element count; entry procedures put and get
2. Specify a **monitor invariant**  
e. g.: **MI**:  $0 \leq \text{length}(\text{buf}) \leq N$
3. Insert **conditional waits**:  
Consider every operation that may violate **MI**, e. g. `enqueue(buf)`;  
find a condition **Cond** such that the operation may be executed safely if **Cond** && **MI** holds,  
e. g. `{ length(buf) < N && MI } enqueue(buf)`;  
define one condition variable **c** for each condition **Cond**  
**insert a conditional wait in front of the operation:**  
`do !(length(buf) < N) -> wait(c); od`  
Loop is necessary in case of **signal-and-continue** or the **may** in step 4!
4. **Insert notification of processes**:  
after every state change that **may** make a waiting condition **Cond** true insert  
`signal(c)` for the condition variable **c** of **Cond**  
e. g. `dequeue(buf); signal(c)`;  
Too many signal calls do not influence correctness - they only cause inefficiency.
5. **Eliminate unnecessary calls of signal** (see PPJ-28)  
Caution: Missing signal calls may cause deadlocks!  
Caution: **signal-and-continue** semantics lacks control of state changes

## Bounded buffers Derivation step 1: monitor **state** and **entry procedures**

```

monitor Buffer
  buf: Queue;                               // state: buf, length(buf)

  init buf = new Queue(n); end
  entry put (d: Data)                        // a producer process tries to store an element

    enqueue (buf, d);

  end;
  entry get (var d: Data)                    // a consumer process tries to take an element

    d := front(buf);
    dequeue(buf);

  end;
end;

```

## Bounded buffers

### Derivation step 2: monitor invariant **MI**

```

monitor Buffer
  buf: Queue; // state: buf, length(buf)

  init buf = new Queue(n); end // MI: 0 <= length(buf) <= N
  entry put (d: Data) // a producer process tries to store an element

    enqueue (buf, d);

  end;
  entry get (var d: Data) // a consumer process tries to take an element

    d := front(buf);
    dequeue(buf);

  end;
end;

```

## Bounded buffers

### Derivation step 3: insert **conditional waits**

```

monitor Buffer
  buf: Queue; // state: buf, length(buf)
  notFull, notEmpty: Condition;
  init buf = new Queue(n); end // MI: 0 <= length(buf) <= N
  entry put (d: Data) // a producer process tries to store an element

    /* length(buf) < N && MI */
    enqueue (buf, d);

  end;
  entry get (var d: Data) // a consumer process tries to take an element

    /* length(buf) > 0 && MI */
    d := front(buf);
    dequeue(buf);

  end;
end;

```

## Bounded buffers

### Derivation step 3: insert **conditional waits**

```

monitor Buffer
  buf: Queue; // state: buf, length(buf)
  notFull, notEmpty: Condition;
  init buf = new Queue(n); end // MI: 0 <= length(buf) <= N
  entry put (d: Data) // a producer process tries to store an element
    do length(buf) >= N -> wait(notFull); od;
    /* length(buf) < N && MI */
    enqueue (buf, d);

  end;

  entry get (var d: Data) // a consumer process tries to take an element
    do length(buf) <= 0 -> wait(notEmpty); od;
    /* length(buf) > 0 && MI */
    d := front(buf);
    dequeue(buf);

  end;
end;

```

## Bounded buffers

### Derivation step 4: insert **notifications**

```

monitor Buffer
  buf: Queue; // state: buf, length(buf)
  notFull, notEmpty: Condition;
  init buf = new Queue(n); end // MI: 0 <= length(buf) <= N
  entry put (d: Data) // a producer process tries to store an element
    do length(buf) >= N -> wait(notFull); od;
    /* length(buf) < N && MI */
    enqueue (buf, d);
    /* length(buf)>0 */
  end;

  entry get (var d: Data) // a consumer process tries to take an element
    do length(buf) <= 0 -> wait(notEmpty); od;
    /* length(buf) > 0 && MI */
    d := front(buf);
    dequeue(buf);
    /* length(buf)<N */
  end;
end;

```



## Bounded buffers

### Derivation step 4: insert **notifications**

```

monitor Buffer
  buf: Queue; // state: buf, length(buf)
  notFull, notEmpty: Condition;
  init buf = new Queue(n); end // MI: 0 <= length(buf) <= N
  entry put (d: Data) // a producer process tries to store an element
    do length(buf) >= N -> wait(notFull); od;
    /* length(buf) < N && MI */
    enqueue (buf, d);
    /* length(buf)>0 */ signal(notEmpty);
  end;
  entry get (var d: Data) // a consumer process tries to take an element
    do length(buf) <= 0 -> wait(notEmpty); od;
    /* length(buf) > 0 && MI */
    d := front(buf);
    dequeue(buf);
    /* length(buf)<N */ signal(notFull);
  end;
end;

```

## Bounded buffers

### Derivation step 5: eliminate **unnecessary notifications**

```

monitor Buffer
  buf: Queue; // state: buf, length(buf)
  notFull, notEmpty: Condition;
  init buf = new Queue(n); end // MI: 0 <= length(buf) <= N
  entry put (d: Data) // a producer process tries to store an element
    do length(buf) >= N -> wait(notFull); od;
    /* length(buf) < N && MI */
    enqueue (buf, d);
    if (length(buf) == 1) signal(notEmpty); // see PPJ-28
    // not correct under signal-and-continue
  end;
  entry get (var d: Data) // a consumer process tries to take an element
    do length(buf) <= 0 -> wait(notEmpty); od;
    /* length(buf) > 0 && MI */
    d := front(buf);
    dequeue(buf);
    if length(buf) == (N-1) -> signal(notFull); // see PPJ-28
    // not correct under signal-and-continue
  end;
end;

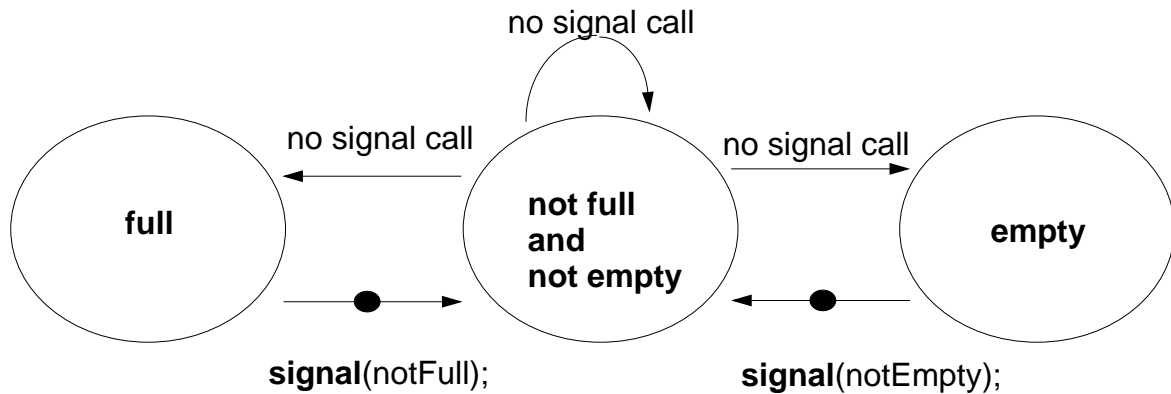
```

## Relevant state changes

Processes need only be awakened when the state change is relevant:  
**when the waiting condition Cond changes from false to true,**  
 i.e. when a waiting process can be released.

These arguments do **not** apply for **signal-and-continue** semantics; there **Cond** may be changed between the signal call and the resume of the released process.

E. g. for the bounded buffer states w.r.t signalling are considered:



## Pattern: Allocating counted resources

A **monitor** grants access to a set of  $k \geq 1$  resources of the **same kind**.  
**Processes** request  $n$  resources,  $1 \leq n \leq k$ , and return them after having used them.  
**Examples:**

- Lending bikes in groups ( $n \geq 1$ ), allocating blocks of storage ( $n \geq 1$ ),
- Taxicab provider ( $n=1$ ), drive with a weight of  $n \geq 1$  tons on a bridge

Monitor invariant	requestRes(1)	returnRes(1)
$0 \leq \text{avail}$ don't give a non-ex. resource	if/do $(!(1 \leq \text{avail}))$ wait(av); avail--;	avail++; /* no wait! */ signal(av);
stronger invariant: $0 \leq \text{avail} \ \&\& \ 0 \leq \text{inUse}$ ... and don't take back more than have been given	if/do $(!(1 \leq \text{avail}))$ wait(av); avail--; inUse++; signal(iu);	if/do $(!(1 \leq \text{inUse}))$ wait(iu); avail++; inUse--; signal(av);
Monitor invariant	requestRes(n)	returnRes(n)
$0 \leq \text{avail}$ don't give a non-ex. resource	do $(!(n \leq \text{avail}))$ wait(av[n]); avail = avail - n;	avail = avail + n; /* no wait! */ signal(av[1]); ... signal(av[avail]);

The **identity** of the resources may be relevant: use a boolean array avail[1] ... avail[k]

## Monitor for resource allocation

A **monitor** grants access to a set of  $k \geq 1$  resources of the **same kind**.

**Processes** request  $n$  resources,  $1 \leq n \leq k$ , and return them after having used them.

Assumption: Process does not return more than it has received  $\Rightarrow$  simpler invariant:

```
class Resources
{ private int avail; // invariant: avail >= 0

  public Resources (int k) { avail = k; }

  synchronized public void getElems (int n) // request n elements
  { while (avail < n) // negated waiting condition
    try { wait(); } catch (InterruptedException e) {}
    avail -= n;
  }

  synchronized public void putElems (int n) // return n elements
  { avail += n; // waiting is not needed because of assumption
    notifyAll(); // notify() would be wrong!
  }
}
```

## Processes and main program for resource monitor

```
import java.util.Random;

class Client extends Thread
{ private Resources mon; private Random rand;
  private int ident, rounds, maximum;

  public Client (Resources m, int id, int rd, int max)
  { mon = m; ident = id; rounds = rd; maximum = max;
    rand = new Random(); // a number generator determines how many
                          // elements are requested in each round,
  }

  public void run () // and when they are returned
  { while (rounds > 0)
    { int m = Math.abs(rand.nextInt()) % maximum + 1;
      mon.getElems (m);
      try { sleep (Math.abs(rand.nextInt()) % 1000 + 1); }
        catch (InterruptedException e) {}
      mon.putElems (m);
      rounds--;
    }
  }
}

public class TestResource
{ public static void main (String[] args)
  { int avail = 20;
    Resources mon = new Resources (avail);
    for (int i=0; i<5; i++)
      new Client (mon, i, 4, avail).start();
  }
}
```

## Readers-Writers problem (Step 1)

A monitor grants reading and writing access to a data base:  
**readers shared, writers exclusive.**

```
monitor ReadersWriters
  nr: int; // number readers
  nw: int; // number writers
  init nr=0; nw=0; end
```

```
entry requestRead()
```

```
  nr++;
```

```
end;
```

```
entry releaseRead()
```

```
  nr--;
```

```
end;
```

```
entry requestWrite()
```

```
  nw++;
```

```
end;
```

```
entry releaseWrite()
```

```
  nw--;
```

```
end;
```

```
end;
```

## Readers-Writers problem (Step 2)

A monitor grants reading and writing access to a data base:  
**readers shared, writers exclusive.**

```
monitor ReadersWriters
  nr: int; // number readers
  nw: int; // number writers
  init nr=0; nw=0; end
```

```
entry requestRead()
```

```
  nr++;
```

```
end;
```

```
entry releaseRead()
```

```
  nr--;
```

```
end;
```

Monitor invariant RW:

$(nr == 0 \parallel nw == 0) \&\& nw \leq 1$

```
entry requestWrite()
```

```
  nw++;
```

```
end;
```

```
entry releaseWrite()
```

```
  nw--;
```

```
end;
```

```
end;
```

## Readers-Writers problem (Step3)

A monitor grants reading and writing access to a data base:  
**readers shared, writers exclusive.**

```
monitor ReadersWriters
  nr: int; // number readers
  nw: int; // number writers
  init nr=0; nw=0; end

  entry requestRead()
    do !(nw==0)
      -> wait(okToRead);
    od;
    { nw==0 && RW }
    nr++;
    { RW }
  end;

  entry releaseRead()
    { RW && nr>0} nr--;

end;
```

Monitor invariant RW:

$$(nr == 0 \parallel nw == 0) \&\& nw \leq 1$$

```
entry requestWrite()
  do !(nr==0 && nw<1)
    -> wait(okToWrite);
  od;
  { nr==0 && nw<1 && RW }
  nw++;
  { RW }
end;

entry releaseWrite()
  { RW && nw==1} nw--;

end;
end;
```

## Readers-Writers problem (Step 4)

A monitor grants reading and writing access to a data base:  
**readers shared, writers exclusive.**

```
monitor ReadersWriters
  nr: int; // number readers
  nw: int; // number writers
  init nr=0; nw=0; end

  entry requestRead()
    do !(nw==0)
      -> wait(okToRead);
    od;
    { nw==0 && RW }
    nr++;
    { RW }
  end;

  entry releaseRead()
    { RW && nr>0} nr--;
    { RW && nr>=0}
    { may be nr==0}

    signal(okToWrite);
  end;

end;
```

Monitor invariant RW:

$$(nr == 0 \parallel nw == 0) \&\& nw \leq 1$$

```
entry requestWrite()
  do !(nr==0 && nw<1)
    -> wait(okToWrite);
  od;
  { nr==0 && nw<1 && RW }
  nw++;
  { RW }
end;

entry releaseWrite()
  { RW && nw==1} nw--;
  { nr==0 && nw==0}
  signal(okToWrite);
  signal_all(okToRead);

end;
end;
```

## Readers-Writers problem (Step 5)

A monitor grants reading and writing access to a data base:  
**readers shared, writers exclusive.**

```
monitor ReadersWriters
  nr: int; // number readers
  nw: int; // number writers
  init nr=0; nw=0; end

entry requestRead()
  do !(nw==0)
    -> wait(okToRead);
  od;
  { nw==0 && RW }
  nr++;
  { RW }
end;

entry releaseRead()
  { RW && nr>0} nr--;
  { RW && nr>=0}
  { may be nr==0}
  if nr==0
    -> signal(okToWrite);
end;
```

Monitor invariant RW:

$$(nr == 0 \parallel nw == 0) \&\& nw \leq 1$$

```
entry requestWrite()
  do !(nr==0 && nw<1)
    -> wait(okToWrite);
  od;
  { nr==0 && nw<1 && RW }
  nw++;
  { RW }
end;

entry releaseWrite()
  { RW && nw==1} nw--;
  { nr==0 && nw==0}
  signal(okToWrite);
  signal_all(okToRead);
end;
end;
```

## Readers/writers monitor in Java

```
class ReaderWriter
{ private int nr = 0, nw = 0;
  // monitor invariant RW: (nr == 0 || nw == 0) && nw <= 1
  synchronized public void requestRead ()
  { while (nw > 0) // negated waiting condition
    try { wait(); } catch (InterruptedException e) {}
    nr++;
  }
  synchronized public void releaseRead ()
  { nr--;
    if (nr == 0) notify (); // awaken one writer is sufficient
  }

  synchronized public void requestWrite ()
  { while (nr > 0 || nw > 0) // negated waiting condition
    try { wait(); } catch (InterruptedException e) {}
    nw++;
  }
  synchronized public void releaseWrite ()
  { nw--;
    notifyAll (); // notify 1 writer and all readers would be sufficient!
  }
}
```

## Method: rendezvous of processes

Processes pass through a **sequence of states** and **interact** with each other.  
A monitor coordinates the **rendezvous in the required order**.

### Design method:

**Specify states by counters;**

characterize **allowed states by invariants** over counters;

**derive waiting conditions** of monitor operations from the invariants;

**substitute counters by binary variables.**

### Example: Sleeping Barber:

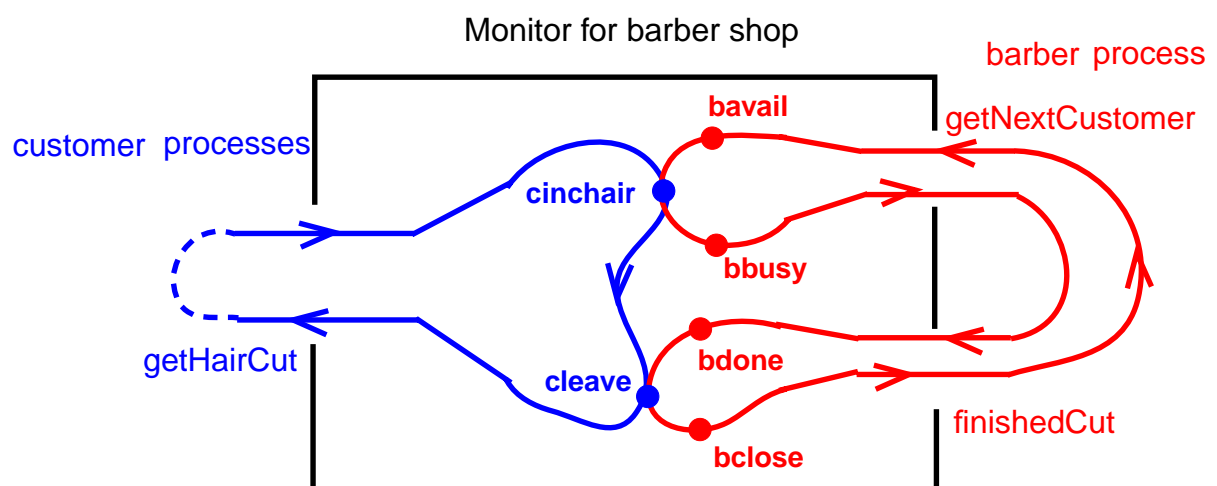
In a sleepy village close to Paderborn a barber is sleeping while waiting for customers to enter his shop. When a customer arrives and finds the barber sleeping, he awakens him, sits in the barber's chair, and sleeps while he gets his hair cut. If the barber is busy when a customer arrives, the customer sleeps in one of the other chairs. After finishing the haircut, the barber gets paid, lets the customer exit, and awakens a waiting customer, if any.

2 kinds of processes: barber (1 instance), customer (many instances)

2 rendezvous: haircut and customer leaves

The task is also an example for the Client/Server pattern.

## Monitor design for the Sleeping Barber problem (step 1)



**Counters** represent states, incremented in entry procedures:

entry proc `getHairCut`:

```
cinchair++;
cleave++;
```

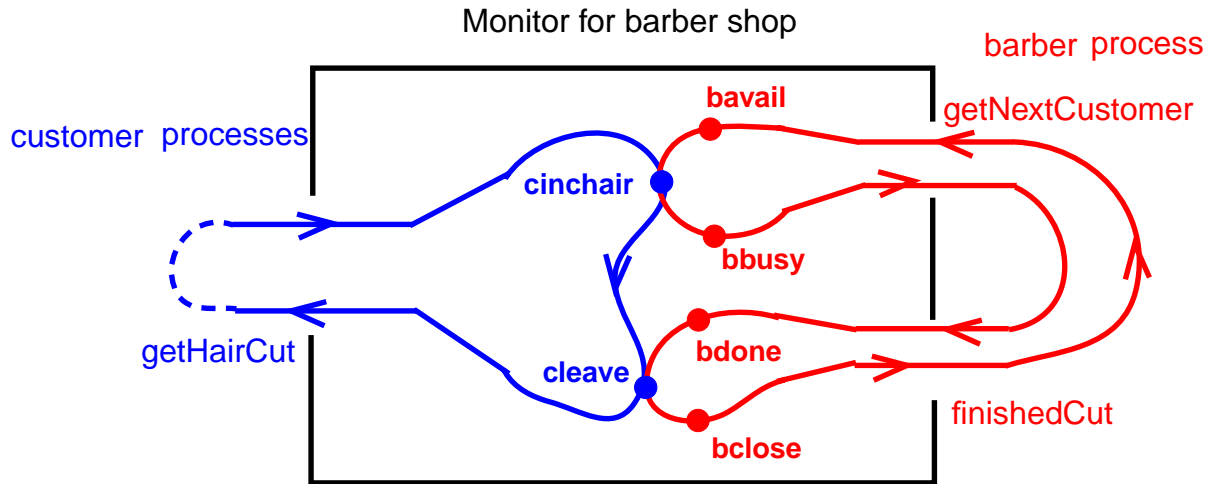
entry proc `getNextCustomer`:

```
bavail++;
bbusy++;
```

entry proc `finishedCut`:

```
bdone++;
bclose++;
```

# Monitor invariant for the Sleeping Barber problem (step 2)



Invariants over counters:

- C1: **cinchair** >= **cleave** and **bavail** >= **bbusy** >= **bdone** >= **bclose**
- C2: **bavail** >= **cinchair** >= **bbusy**
- C3: **bdone** >= **cleave** >= **bclose**

Monitor invariant: BARBER: C1 and C2 and C3

# Waiting conditions for the Sleeping Barber problem (step 3)

Monitor invariant: BARBER: C1 and C2 and C3:

- C1: **cinchair** >= **cleave** and **bavail** >= **bbusy** >= **bdone** >= **bclose**
- C2: **bavail** >= **cinchair** >= **bbusy**
- C3: **bdone** >= **cleave** >= **bclose**

guaranteed by execution order  
 leads to 2 waiting conditions  
 leads to 2 waiting conditions

entry proc **getHairCut**:

```
do not (bavail > cinchair) -> wait (b); done;
cinchair++;
do not (bdone > cleave) -> wait (o); done;
cleave++;
```

entry proc **getNextCustomer**:

```
bavail++;
do not (cinchair > bbusy) -> wait (c); done;
bbusy++;
```

entry proc **finishedCut**:

```
bdone++;
do not (cleave > bclose) -> wait (e); done;
bclose++;
```



## Substitute counters (step 3a)

new binary variables:

**barber** = **bavail** - **cinchair**  
**chair** = **cinchair** - **bbusy**  
**open** = **bdone** - **cleave**  
**exit** = **cleave** - **bclose**

value ranges: {0, 1}

Old invariants:

C2: **bavail** >= **cinchair** >= **bbusy**  
C3: **bdone** >= **cleave** >= **bclose**

New invariants:

C2: **barber** >= 0 && **chair** >= 0  
C3: **open** >= 0 && **exit** >= 0

increment operations and conditions are substituted:

entry proc **getHairCut**:

do not (**barber** > 0) -> wait (**b**); done;  
**barber--**; **chair++**;  
do not (**open** > 0) -> wait (**o**); done;  
**open--**; **exit++**;

entry proc **getNextCustomer**:

**barber++**;  
do not (**chair** > 0) -> wait (**c**); done;  
**chair--**;

entry proc **finishedCut**:

**open++**;  
do not (**exit** > 0) -> wait (**e**); done;  
**exit--**;

## Signal operations for the Sleeping Barber problem (step 4)

new binary variables:

**barber** = **bavail** - **cinchair**  
**chair** = **cinchair** - **bbusy**  
**open** = **bdone** - **cleave**  
**exit** = **cleave** - **bclose**

value ranges: {0, 1}

Old invariants:

C2: **bavail** >= **cinchair** >= **bbusy**  
C3: **bdone** >= **cleave** >= **bclose**

New invariants:

C2: **barber** >= 0 && **chair** >= 0  
C3: **open** >= 0 && **exit** >= 0

insert call signal (x) call where a condition of x may become true:

entry proc **getHairCut**:

do not (**barber** > 0) -> wait (**b**); done;  
**barber--**; **chair++**; **signal (c)**;  
do not (**open** > 0) -> wait (**o**); done;  
**open--**; **exit++**; **signal (e)**;

entry proc **getNextCustomer**:

**barber++**; **signal (b)**;  
do not (**chair** > 0) -> wait (**c**); done;  
**chair--**;

entry proc **finishedCut**:

**open++**; **signal (o)**;  
do not (**exit** > 0) -> wait (**e**); done;  
**exit--**;

## 5. Data Parallelism: Barriers

Many processes execute the **same operations at the same time on different data**; usually on elements of **regular data structures**: arrays, sequences, matrices, lists.

Data parallelism as an **architectural model of parallel computers**:

**vector machines**, e. g. Cray

**SIMD machines** (Single Instruction Multiple Data), e. g. Connection Machine, MasPar  
GPUs (Graphical Processing Units); massively parallel processors on graphic cards

Data parallelism as a **programming model for parallel computers**:

- computations on **arrays in nested loops**
- analyze **data dependences** of computations, **transform** and **parallelize** loops
- iterative **computations in rounds**, synchronize with **Barriers**
- **systolic computations**: 2 phases are iterated: compute - shift data to neighbour processes

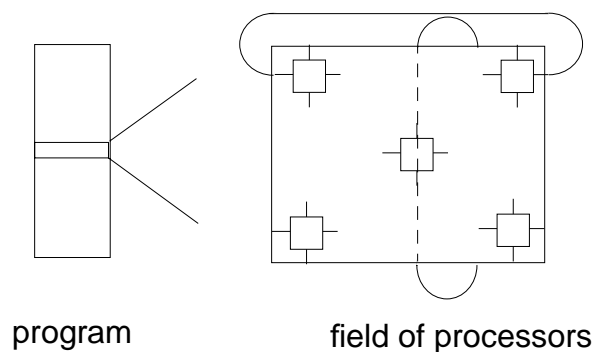
**Applications** mainly in **technical, scientific computing**, e. g.

- fluid mechanics
- image processing
- solving differential equations
- finite element method in design systems

## Data parallelism as an architectural model

**SIMD** machine: Single Instruction Multiple Data

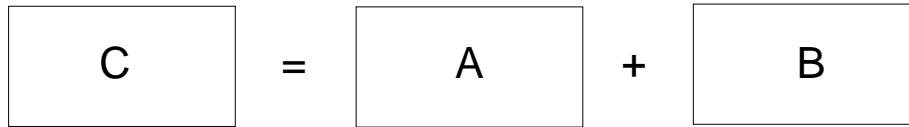
- very many processors, **massively parallel**  
e. g. 32 x 64 processor field
- **local memory** for each processor
- same instructions in **lock step**
- fast communication in **lock step**
- fixed topology, usually a **grid**
- machine types e. g. Connection Machine, MasPar



## Data parallelism as a programming model

- regular data structures (arrays, lists) are mapped onto a field of processors
- processes execute the same program on individual data in lock step
- communication with neighbours in the same direction in lock step

simple example matrix addition:



sequential:

```
for (i = 0; i < N; i++)
  for (j = 0; j < M; j++)
    c[i,j] = a [i,j] + b[i,j];
```

```
distribute A, B
c = a + b
collect C
```

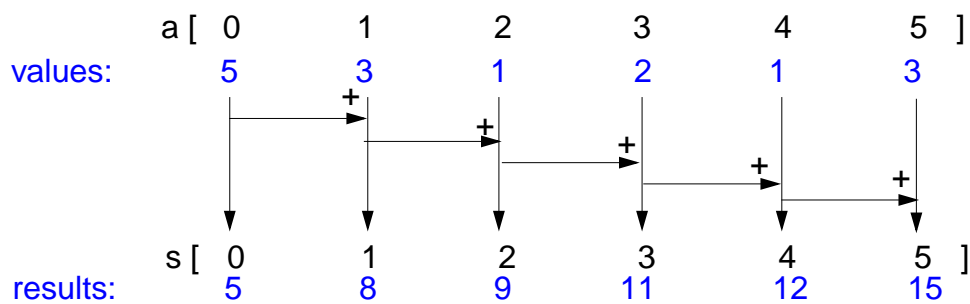
1 step!

- these can be parallelized directly, since there are no **data dependences**
- **data mapping** is trivial: array element [i,j] on process [i,j]
- **communication** is not needed
- no **algorithmic idea** is needed

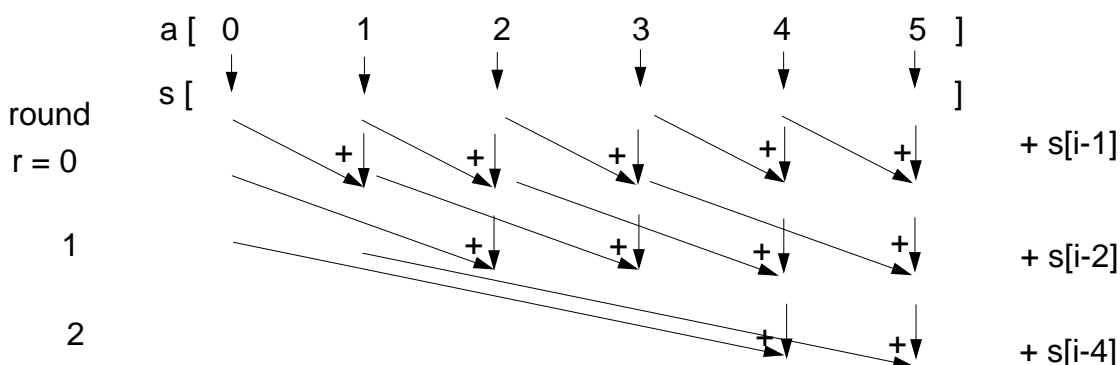
## Example prefix sums

input: sequence a of numbers;  
output: sequence s of sums of the prefixes of a

$$s[i] = \sum_{j=0}^i a[j]$$



parallel algorithmic idea:

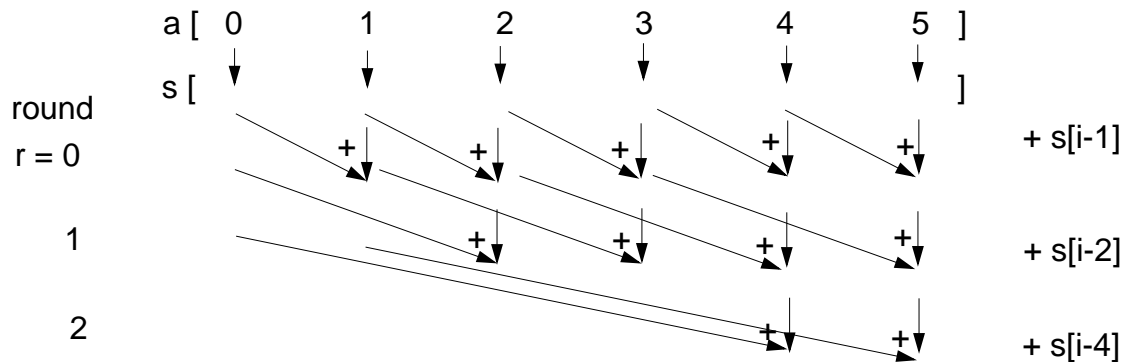


## Example prefix sums (2)

input: sequence  $a$  of numbers;  
output: sequence  $s$  of sums of the prefixes of  $a$

$$s[i] = \sum_{j=0}^i a[j]$$

parallel algorithmic idea:



**Proof for process  $p = 0 \dots n - 1$**

**Invariant SUM:**  $s[p] = a[p-d+1] + \dots + a[p]$  with  $d = 1, 2, \dots, m \leq n$  distance before next round

**Induction begin:**  $d = 1$ ;  $s[p] = a[p]$  holds by initialization

**induction step:** computation  $s[p] = s[p - d] + a[p-2d+1] + \dots + a[p-d] + a[p-d+1] + \dots + a[p]$

substitution of  $2d$  by  $d$  implies SUM

## Prefix sums: applied methods

- computational scheme **reduction**:  
all array elements are comprised using a reduction operation (here: addition)
- iterative **computation in rounds**:  
in each round all processes perform a computation step
- **duplication of distance**:  
data is exchanged in each round with a neighbour at twice the distance as in the previous round
- **barrier** synchronization:  
processes may not enter the next round, before all processes have finished the previous one

## Barriers

Several processes meet at a common point of synchronization

**Rule:** All processes must have reached the barrier (for the j-th time), before one of them leaves it (for the j-th time).

### Applications:

- iterative computations, where iteration j uses results of iteration j-1
- separation of computational phases

### Scheme:

```
public void run ()
{ do { computeNewValues (i);
      b.barrier();
    }
  while (!converged);
}
```

### Implementation techniques for barriers:

- central controller: monitor or coordination process
- worker processes coordinated as a tree
- worker processes symmetrically coordinated (butterfly barrier, dissemination barrier)

## Barrier implemented by a monitor

Monitor stops a given number of processes and releases them together:

```
class BarrierMonitor
{ private int  processes           // number of processes to be synchronized
      arrived = 0;               // number of processes arrived at the barrier

  public BarrierMonitor (int procs)
  { processes = procs; }

  synchronized public barrier ()
  { arrived++;
    if (arrived < processes)
      try { wait(); } catch (InterruptedException e) {}
                                     // exception destroys barrier behaviour

    else
    { arrived = 0;                       // reset arrival count
      notifyAll();                       // release the other processes
    } } }
```

## Distributed tree barrier

Barrier synchronization of the worker processes organized as a **binary tree**.  
Bottleneck of central synchronization is avoided.

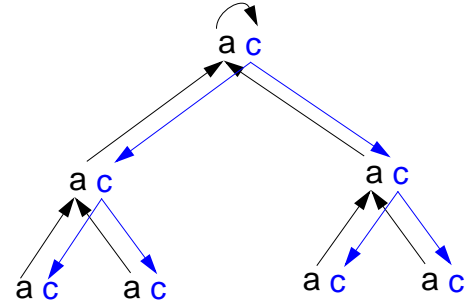
### 2 synchronization variables (flags) at each node:

**arrived:** all processes in a subtree have arrived, is propagated upward

**continue:** all processes in a subtree may continue, is propagated downward

disadvantage:

**different code** is needed for root, inner nodes, and for leafs



## 2 Rules for Synchronization Using Flags

Flag for synchronization between exactly 2 processes

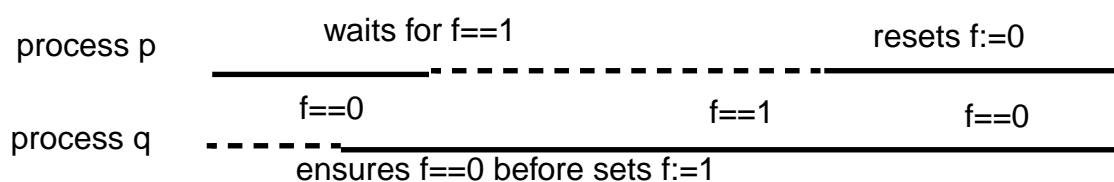
One process waits until the flag is set.  
The other process sets the flag.

May be implemented by a monitor in Java.

**Flag rules:**

1. The process that waits for a flag resets it.
2. A flag that is set may not be set again before being reset.

Consequence: no state change will be lost.



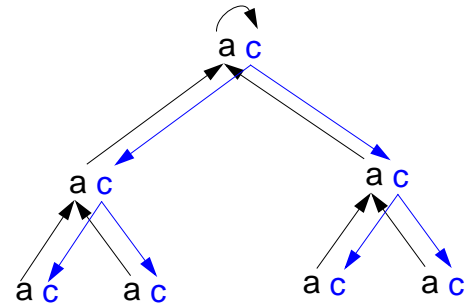
## Distributed tree barrier implementation

### 2 synchronization variables (flags) at each node:

arrived: all processes in a subtree have arrived propagated upward

continue: all processes in a subtree may continue propagated downward

initially all flags are reset



code for an **inner** node:

```
execute this.task();
wait for left.arrived; reset left.arrived;
wait for right.arrived; reset right.arrived;
set this.arrived;
wait for this.continue; reset this.continue;
set left.continue;
set right.continue;
```

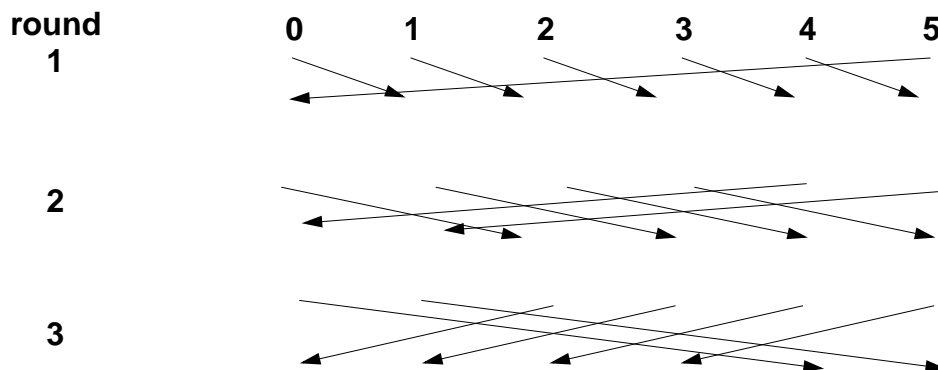
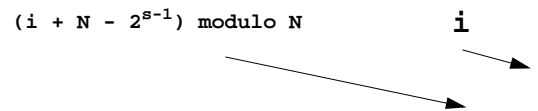
	leaf	root
execute this.task();	x	x
wait for left.arrived; reset left.arrived;		x
wait for right.arrived; reset right.arrived;		x
set this.arrived;	x	
wait for this.continue; reset this.continue;	x	
set left.continue;		x
set right.continue;		x

## Symmetric, distributed barrier (dissemination)

Processes **synchronize pairwise** in rounds with doubled distances.

N processes are synchronized after r rounds if  $N \leq 2^r$

In round s  
process i indicates its arrival and then waits  
for the arrival of process  $(i + N - 2^{s-1}) \text{ modulo } N$ :



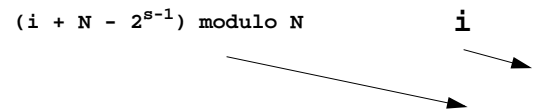
After r rounds each process is synchronized with each other. Proof idea: For each process i each other process occurs in a tree of processes which have synchronized (in)directly with i.

## Symmetric, distributed barrier: implementation

In round  $s$

process  $i$  indicates its arrival and

waits for the arrival of process  $(i + N - 2^{s-1}) \text{ modulo } N$



Code for each process:

```
execute this.task();

// synchronize:
s = 0;
while (N > 2s)
    s++;
    wait for f==0; set f=1;
    partner=p[(i + N - 2s-1) modulo N];
    wait partner.f; reset partner.f=0
```

## Prefix sums with barriers

```
class PrefixSum extends Thread
{ private int procNo; // number of process
  private BarrierMonitor bm; // barrier object

  public PrefixSum (int p, BarrierMonitor b)
  { procno = p; bm = b; }

  public void run ()
  { int addIt, dist = 1; // distance
    // global arrays a and s
    // initialize result array
    s[procNo] = a[procNo];
    bm.barrier();

    // invariant SUM: s[procNo] == a[procNo-dist+1]+...+a[procNo]
    while (dist < N)
    { if (procNo - dist >= 0)
      addIt = s[procNo - dist]; // value before overwritten
      bm.barrier();
      if (procNo - dist >= 0)
        s[procNo] += addIt;
      bm.barrier();
      dist = dist * 2; // doubled distance
    } } }
```



## Prefix sums in a synchronous parallel programming model

Notation in Modula-2\* with synchronous (and asynchronous) loops for parallel machines

```

VAR a, s, t: ARRAY [0..N-1] OF INTEGER;
VAR dist: CARDINAL;
BEGIN
  ...
  FORALL i: [0..N-1] IN SYNC                parallel loop in lock step
    s[i] := a[i];
  END;

  dist := 1;

  WHILE dist < N                            parallel loop in lock step
    FORALL i: [0..N-1] IN SYNC
      IF (i-dist) >= 0 THEN
        t[i] := s[i - dist];
        s[i] := s[i] + t[i];                implicit barrier
                                           for each statement
      END
    END;
    dist := dist * 2;
  END
END

```

## Finding list ends: data parallel approach

input: int array link stores lists; link[i] contains the index of the successor or nil

output: int array last; last[i] contains the index of the last element of list link[i]

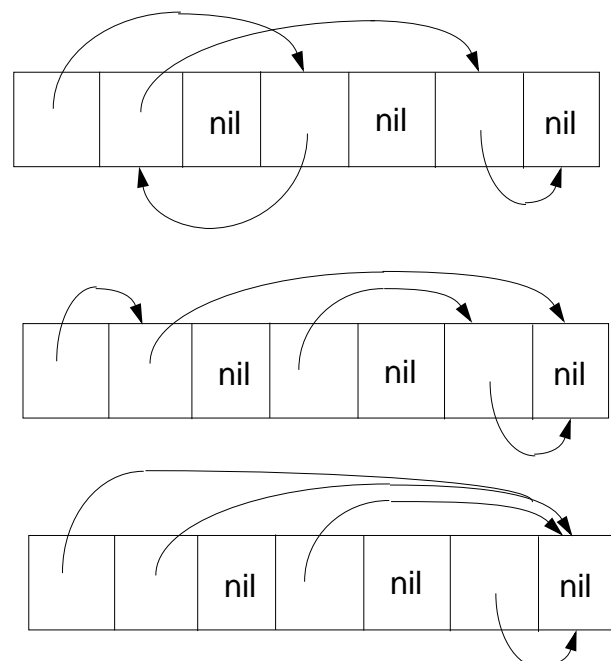
method: **worker process** i computes  $\text{last}[i] = \text{last}[\text{last}[i]]$  in log N rounds

```

int d = 1;
last[i] = link[i];
barrier
while (d < N)
{
  int newlast = nil;
  if ( last[i] != nil &&
      last[last[i]] != nil)
    newlast = last[last[i]];
  barrier
  if (newlast != nil)
    last[i] = newlast;
  barrier
  d = 2*d;
}

```

last[i] points to the end of those lists which are not longer than d



## 5.2 / 6. Data Parallelism: Loop Parallelization

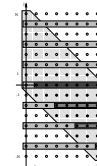
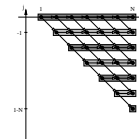
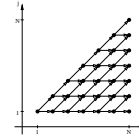
**Regular loops** on orthogonal data structures - parallelized for **data parallel** processors

Development steps (automated by compilers):

- **nested loops** operating on **arrays**, sequential execution of iteration space
- analyze **data dependences**  
data-flow: definition and use of array elements
- **transform loops**  
keep data dependences forward in time
- **parallelize inner loop(s)**  
map to field or vector of processors
- **map arrays to processors**  
such that many accesses are local,  
transform index spaces

```

DECLARE B[0..N,0..N+1]
FOR I := 1 .. N
  FOR J := 1 .. I
    B[I,J] :=
      B[I-1,J]+B[I-1,J-1]
  END FOR
END FOR
  
```



## Iteration space of loop nests

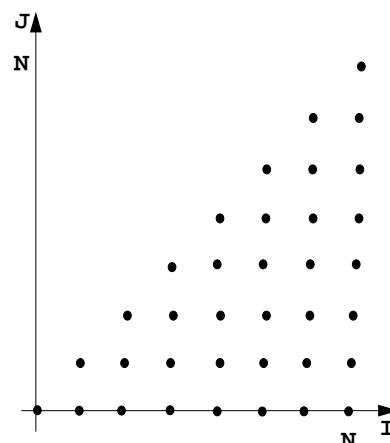
**Iteration space** of a loop nest of depth n:

- **n-dimensional space of integral points** (polytope)
- each point  $(i_1, \dots, i_n)$  represents an execution of the innermost loop body
- loop bounds are in general not known before run-time
- iteration need not have orthogonal borders
- iteration is elaborated sequentially

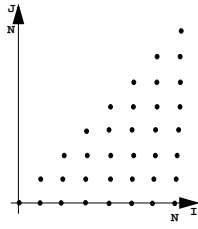
example:  
computation of Pascal's triangle

```

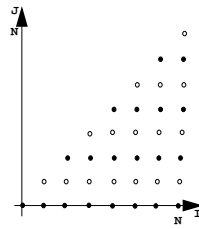
DECLARE B[-1..N,-1..N]
FOR I := 0 .. N
  FOR J := 0 .. I
    B[I,J] :=
      B[I-1,J]+B[I-1,J-1]
  END FOR
END FOR
  
```



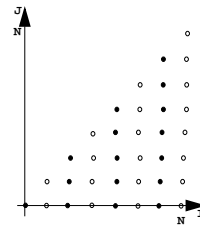
## Examples for Iteration spaces of loop nests



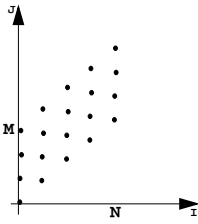
```
FOR I := 0 .. N
  FOR J := 0 .. I
```



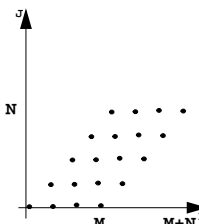
```
FOR I := 0 .. N
  FOR J := 0..I BY 2
```



```
FOR I := 0..N BY 2
  FOR J := 0 .. I
```



```
FOR I := 0 .. N
  FOR J := I..I+M
M = 3, N = 4
```



```
FOR I := 0 .. M+N
  FOR J := max(0, I-M)..
    min(I, N)
```

## Data Dependences in Iteration Spaces

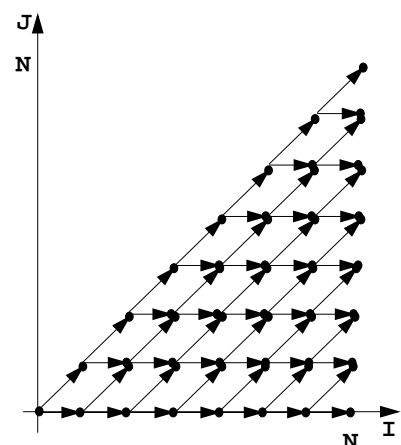
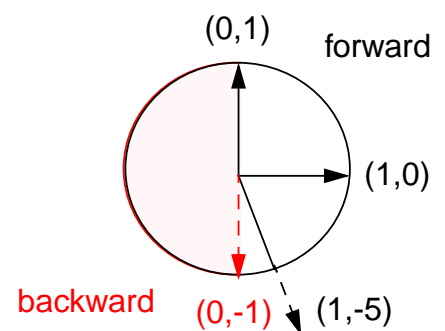
### Data dependence from iteration point $i_1$ to $i_2$ :

- Iteration  $i_1$  computes a value that is used in iteration  $i_2$  (flow dependence)
- relative **dependence vector**  
 $\mathbf{d} = \mathbf{i}_2 - \mathbf{i}_1 = (i_{2_1} - i_{1_1}, \dots, i_{2_n} - i_{1_n})$   
 holds for all iteration points except at the border
- Flow-dependences can **not be directed against the execution order**, can not point backward in time: each dependence vector must be **lexicographically positive**, i. e.  $\mathbf{d} = (0, \dots, 0, d_j, \dots)$ ,  $d_j > 0$

Example:

Computation of Pascal's triangle

```
DECLARE B[-1..N,-1..N]
FOR I := 0 .. N
  FOR J := 0 .. I
    B[I,J] :=
      B[I-1,J]+B[I-1,J-1]
  END FOR
END FOR
```



# Loop Transformation

The **iteration space** of a loop nest is transformed to **new coordinates**. Goals:

- **execute innermost loop(s) in parallel**
- improve **locality** of data accesses;  
**in space**: use storage of executing processor,  
**in time**: reuse values stored in cache
- **systolic** computation and communication scheme

Data dependences must **point forward in time**, i.e. **lexicographically positive** and **not within parallel dimensions**

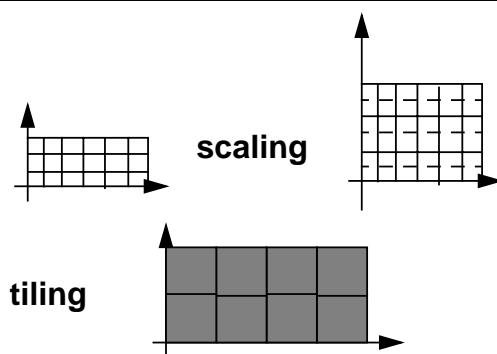
**linear basic transformations:**

- **Skewing**: add iteration count of an outer loop to that of an inner one
- **Reversal**: flip execution order for one dimension
- **Permutation**: exchange two loops of the loop nest

**SRP transformations** (next slides)

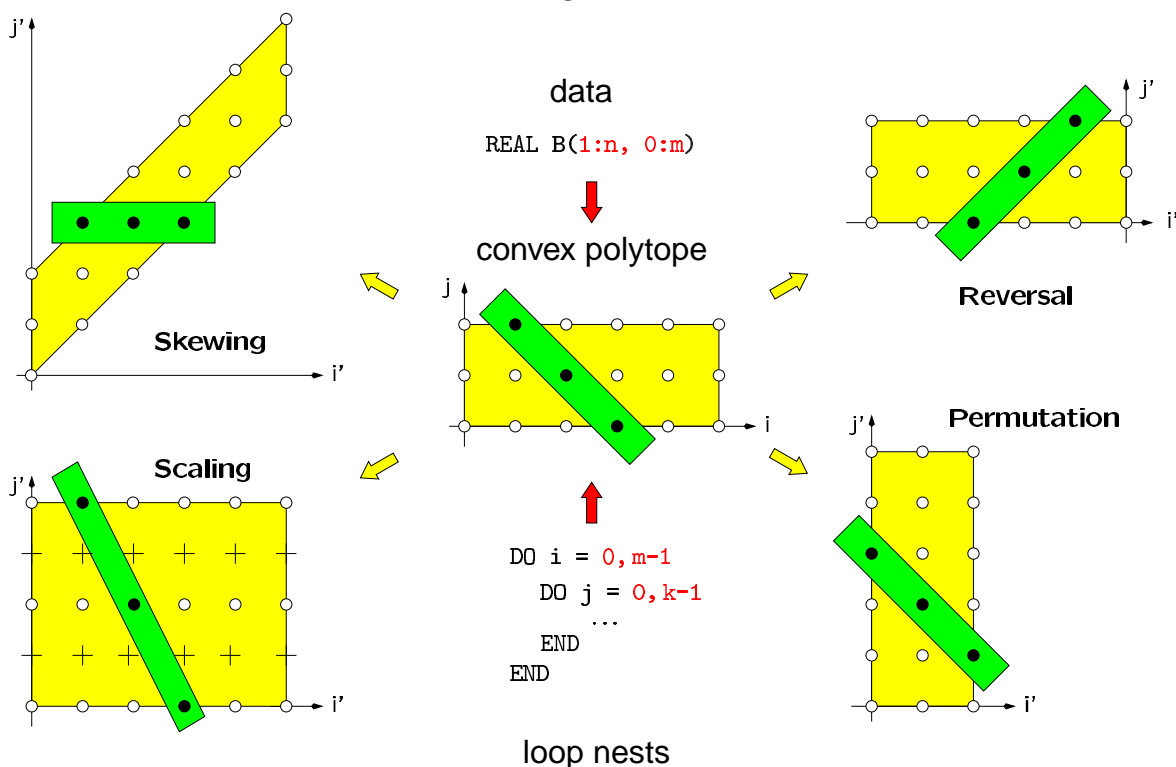
**non-linear transformations**, e. g.

- **Scaling**: stretch the iteration space in one dimension, causes gaps
- **Tiling**: introduce **additional inner loops** that **cover tiles** of fixed size



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## Transformations of



## Transformations defined by matrices

Transformation matrices: systematic transformation, check dependence vectors

$$\text{Reversal} \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} i \\ -j \end{pmatrix} = \begin{pmatrix} i' \\ j' \end{pmatrix}$$

$$\text{Skewing} \quad \begin{pmatrix} 1 & 0 \\ f & 1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} i \\ f*i+j \end{pmatrix} = \begin{pmatrix} i' \\ j' \end{pmatrix}$$

$$\text{Permutation} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} j \\ i \end{pmatrix} = \begin{pmatrix} i' \\ j' \end{pmatrix}$$

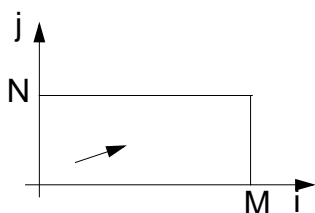
## Reversal

Iteration count of one loop is negated, that dimension is enumerated backward

general transformation matrix

$$\begin{pmatrix} 1 & & & & 0 \\ \dots & & & & \\ & 1 & & & \\ 0 & -1 & & & \\ & & 1 & & \dots \\ & & & & & 1 \end{pmatrix}$$

```
for i = 0 to M
  for j = 0 to N
    ...
```



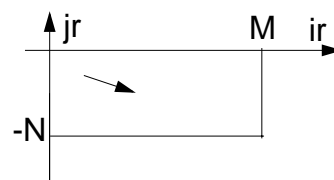
2-dimensional:

loop variables  
old                      new

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} i \\ -j \end{pmatrix} = \begin{pmatrix} i_r \\ j_r \end{pmatrix}$$

```
for i_r = 0 to M
  for j_r = -N to 0
    ...
```

original  
transformed



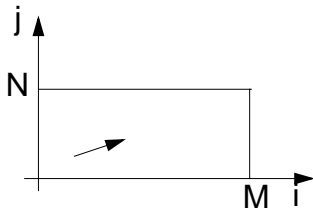
## Skewing

The **iteration count** of an outer loop is **added to the count of an inner loop**;  
iteration space is shifted; **execution order** of iteration points **remains unchanged**

general transformation matrix:

$$\begin{pmatrix} 1 & & & & \\ & \dots & & & 0 \\ & & 1 & & \\ & f & 1 & & \\ & & & 1 & \dots \\ & 0 & & & & 1 \end{pmatrix}$$

```
for i = 0 to M
  for j = 0 to N
    ...
```



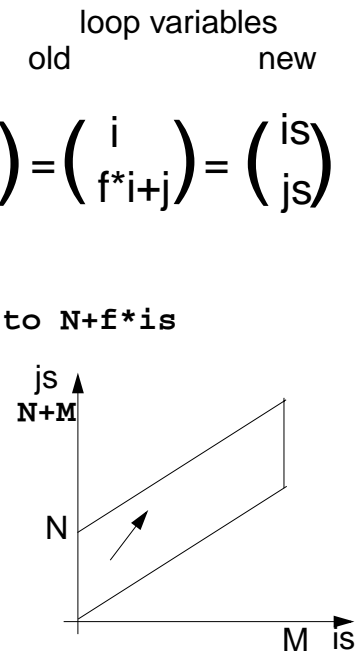
original

2-dimensional:

$$\begin{pmatrix} 1 & 0 \\ f & 1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} i \\ f*i+j \end{pmatrix} = \begin{pmatrix} is \\ js \end{pmatrix}$$

```
for is = 0 to M
  for js = f*is to N+f*is
    ...
```

transformed



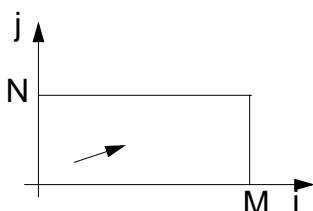
## Permutation

**Two loops of the loop nest are interchanged**; the iteration space is flipped;  
the **execution order** of iteration points **changes**; new dependence vectors must be legal.

general transformation matrix:

$$\begin{pmatrix} 1 & & & & \\ & 0 & 1 & & 0 \\ & & & 1 & \\ & 1 & 0 & & \\ & 0 & & & 1 & \dots \\ & & & & & & 1 \end{pmatrix}$$

```
for i = 0 to M
  for j = 0 to N
    ...
```



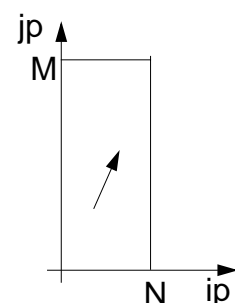
original

2-dimensional:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} j \\ i \end{pmatrix} = \begin{pmatrix} ip \\ jp \end{pmatrix}$$

```
for ip = 0 to N
  for jp = 0 to M
    ...
```

transformed



## Use of Transformation Matrices

- Transformation matrix  $T$  defines **new iteration counts** in terms of the old ones:  $T * i = i'$

e. g. Reversal 
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} i \\ -j \end{pmatrix} = \begin{pmatrix} i' \\ j' \end{pmatrix}$$

- Transformation matrix  $T$  transforms old **dependence vectors** into new ones:  $T * d = d'$

e. g. 
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} * \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

- inverse Transformation matrix  $T^{-1}$  defines **old iteration counts** in terms of new ones, for transformation of index expressions in the loop body:  $T^{-1} * i' = i$

e. g. 
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} * \begin{pmatrix} i' \\ j' \end{pmatrix} = \begin{pmatrix} i' \\ -j' \end{pmatrix} = \begin{pmatrix} i \\ j \end{pmatrix}$$

- concatenation of transformations** first  $T_1$  then  $T_2$ :  $T_2 * T_1 = T$

e. g. 
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

## Inequalities Describe Loop Bounds

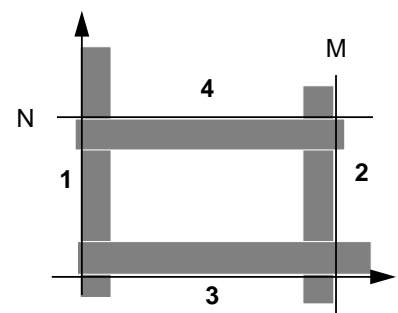
The bounds of a loop nest are described by a **set of linear inequalities**.  
Each **inequality separates the space** in „inside and outside of the iteration space“:

$$B * i \leq c$$

$$\begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} \leq \begin{pmatrix} 0 \\ M \\ 0 \\ N \end{pmatrix}$$

example 1

- $-i \leq 0$
- $i \leq M$
- $-j \leq 0$
- $j \leq N$

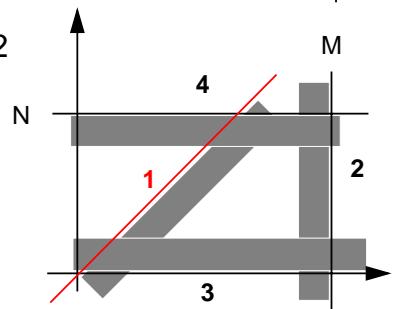


$$\begin{pmatrix} -1 & 1 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} \leq \begin{pmatrix} 0 \\ M \\ 0 \\ N \end{pmatrix}$$

example 2

- $-i + j \leq 0$
- $i \leq M$
- $-j \leq 0$
- $j \leq N$

transformed



**positive** factors represent **upper** bounds  
**negative** factors represent **lower** bounds

$$1, 4: j \leq \min(i, N)$$

$$3: 0 \leq j$$

$$1 + 3: 0 \leq i$$

$$2: i \leq M$$

## Transformation of Loop Bounds

The inverse of a transformation matrix  $T^{-1}$  transforms a set of inequalities:  $B * T^{-1} i' \leq c$

$$\begin{matrix} \text{skewing} & \text{inverse} & B & T^{-1} & B * T^{-1} \\ \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} & \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} & * \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} & = \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ -1 & 1 \end{pmatrix} \end{matrix}$$

example 1  
new bounds:

$$\begin{matrix} B * T^{-1} & i' & c \\ \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ -1 & 1 \end{pmatrix} & * \begin{pmatrix} i' \\ j' \end{pmatrix} & \leq \begin{pmatrix} 0 \\ M \\ 0 \\ N \end{pmatrix} \end{matrix}$$

1  $-i' \leq 0$   
 2  $i' \leq M$   
 3  $i' - j' \leq 0$   
 4  $-i' + j' \leq N$

## Example for Transformation and Parallelization of a Loop

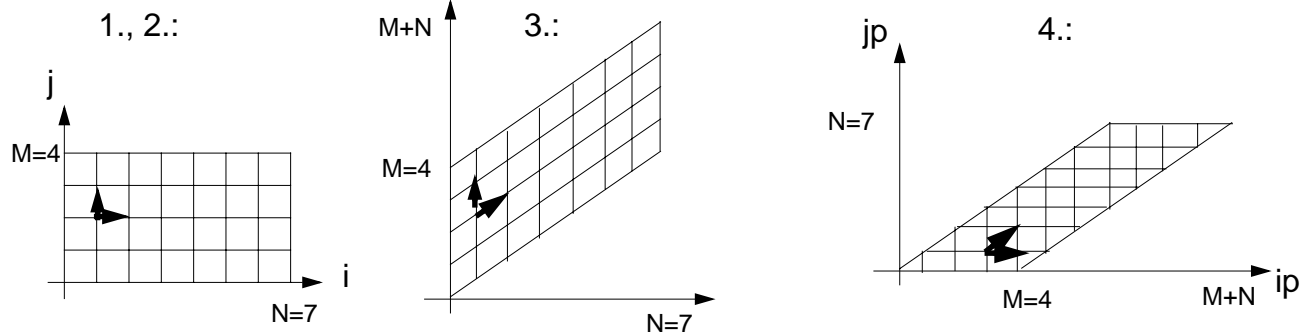
```
for i = 0 to N
  for j = 0 to M
    a[i, j] = (a[i, j-1] + a[i-1, j]) / 2;
```

Parallelize the above loop.

1. Draw the iteration space.
2. Compute the dependence vectors and draw examples of them into the iteration space. Why can the inner loop not be executed in parallel?
3. Apply a skewing transformation and draw the iteration space.
4. Apply a permutation transformation and draw the iteration space. Explain why the inner loop now can be executed in parallel.
5. Compute the matrix of the composed transformation and use it to transform the dependence vectors.
6. Compute the inverse of the transformation matrix and use it to transform the index expressions.
7. Specify the loop bounds by inequalities and transform them by the inverse of the transformation matrix.
8. Write the complete loops with new loop variables  $i_p$  and  $j_p$  and new loop bounds.



## Solution of the Transformation and Parallelization Example



5.:

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

6.: Inverse

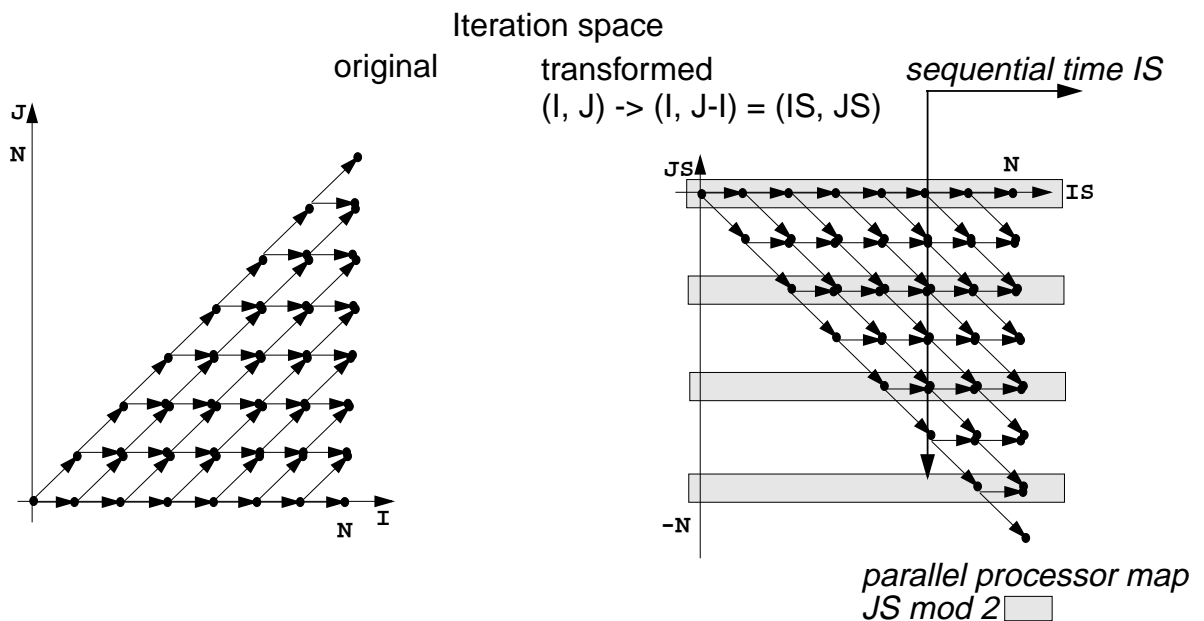
$$\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$$

7. Bounds:

	$B$	$c$	$B * T^{-1}$		
orig.:	$\begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ N \\ 0 \\ M \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ 0 & 1 \\ -1 & 1 \\ 1 & -1 \end{pmatrix}$	1 $-jp \leq 0$	1, 3 $\Rightarrow 0 \leq ip$
				2 $jp \leq N$	2, 4 $\Rightarrow ip \leq M+N$
				3 $-ip+jp \leq 0$	1, 4 $\Rightarrow \max(0, ip-M) \leq jp$
				4 $ip - jp \leq M$	2, 3 $\Rightarrow jp \leq \min(ip, N)$

8. for  $ip = 0$  to  $M+N$   
 for  $jp = \max(0, ip-M)$  to  $\min(ip, N)$   
 $a[jp, ip-jp] = (a[jp, ip-jp-1] + a[jp-1, ip-jp]) / 2;$

## Transformation and Parallelization



```

DECLARE B[-1..N,-1..N]
FOR I := 0 .. N
  FOR J := 0 .. I
    B[I,J] :=
      B[I-1,J]+B[I-1,J-1]
  END FOR
END FOR

```

```

DECLARE B[-1..N,-1..N]
FOR IS := 0.. N
  FOR JS := -IS .. 0
    B[IS,JS+IS] :=
      B[IS-1,JS+IS]+B[IS-1,JS-1+IS]
  END FOR
END FOR

```

# Data Mapping

## Goal:

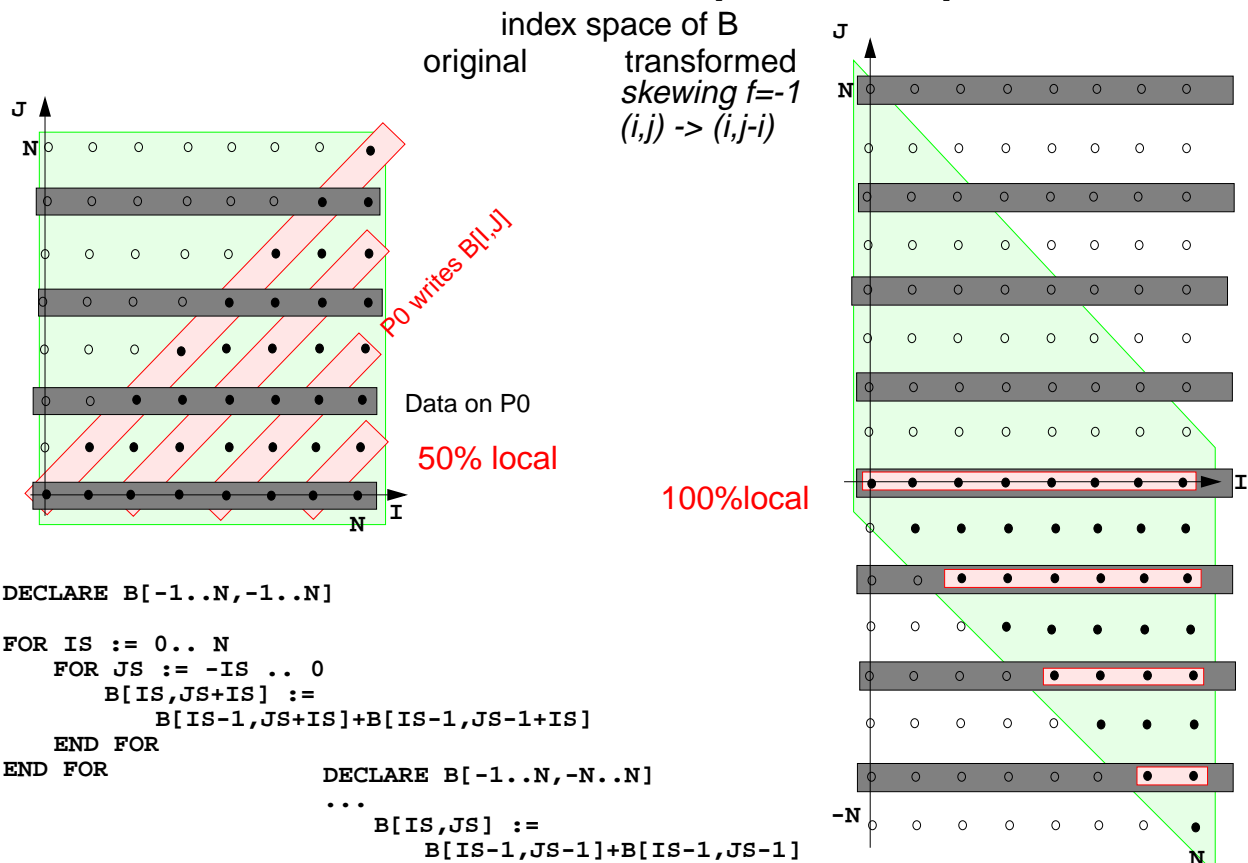
**Distribute array elements** over processors, such that as many **accesses as possible** are local.

## Index space of an array:

n-dimensional space of integral index points (polytope)

- **same properties as iteration space**
- same mathematical model
- same **transformations** are applicable (Skewing, Reversal, Permutation, ...)
- **no restrictions** by data dependences

## Data distribution for parallel loops

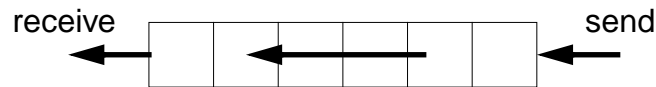


## 7. Asynchronous Message Passing

Processes send and receive messages via channels

**Message:** value of a composed data type or object of a class

**Channel:** **queue** of arbitrary length, containing messages



**operations** on a channel:

- **send (b):** adds the message b to the end of the queue of the channel; does **not block** the executing process (in contrast to synchronous send)
- **receive():** yields the oldest message and deletes it from the channel; **blocks** the executing process as long as the channel is empty.
- **empty():** yields true, if the channel is empty; the result is **not necessarily up-to-date**.

**send** and **receive** are executed under mutual exclusion.

## Channels implemented in Java

```
public class Channel
{
    // implementation of a channel using a queue of messages
    private Queue msgQueue;

    public Channel ()
    { msgQueue = new Queue (); }

    public synchronized void send (Object msg)
    { msgQueue.enqueue (msg); notify(); } // wake a receiving process

    public synchronized Object receive ()
    { while (msgQueue.empty())
      try { wait(); } catch (InterruptedException e) {}
      Object result = msgQueue.front(); // the queue is not empty
      msgQueue.dequeue();
      return result;
    }

    public boolean empty ()
    { return msgQueue.empty (); }
}
```

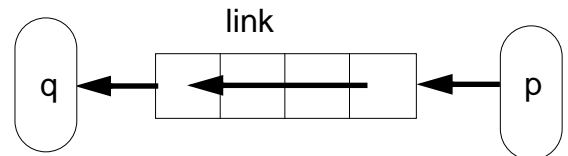
All waiting processes wait for the same condition => notify() is sufficient.

After a notify-call a new receive-call may have stolen the only message => wait loop is needed

## Processes and channels

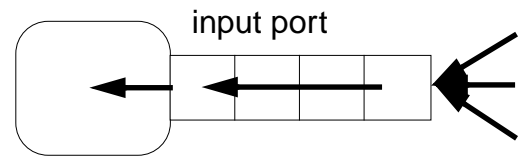
### link:

**one sender** is connected to **one receiver**;  
e. g. processes form chains of transformation steps (pipeline)



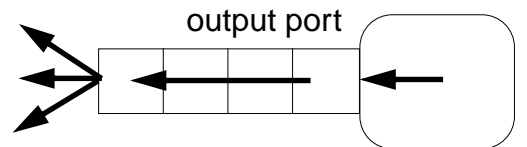
### input port of a process:

**many senders - one receiver**;  
channel belongs to the receiving process;  
e. g. a server process receives tasks from several client processes



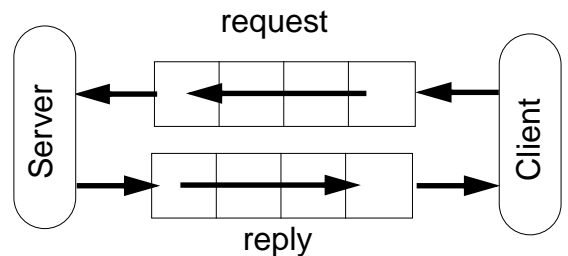
### output port of a process:

**one sender - many receivers**;  
channel belongs to the sending process;  
e. g. a process distributes tasks to many servers (unusual structure)



### pair of request and reply channels;

one process requests - the others replies;  
tight synchronization,  
e. g. between client and server



## Termination conditions

When system of processes terminates the following **conditions** must hold:

1. **All channels are empty.**
2. **No processes are blocked on a receive operation.**
3. **All processes are terminated.**

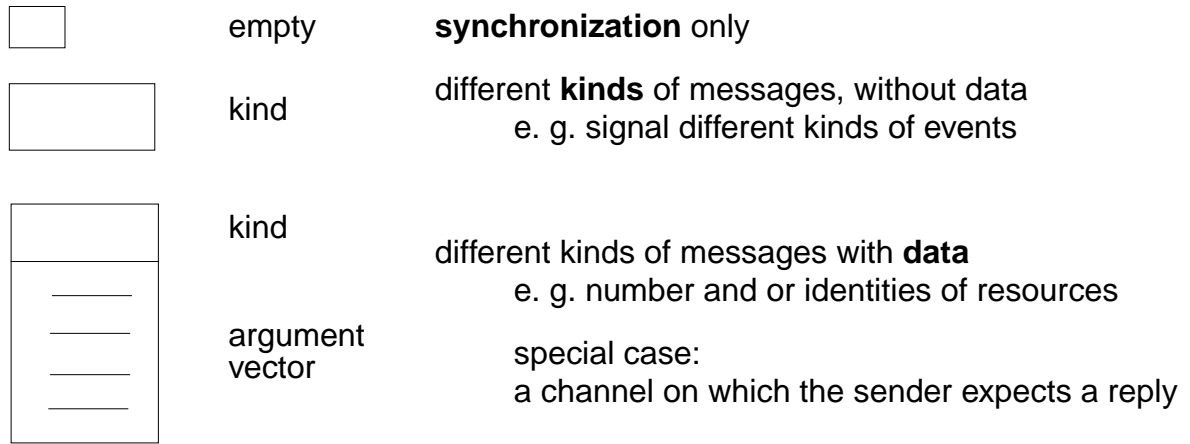
Otherwise the **system state is not well-defined**, e.g. task is not completed, some operations are pending.

### Problem:

In general, the processes **do not know the global system state**.

# Message structures

A **message object** may have arbitrary structure suitable for the **particular purpose**:



**Operations** on messages:

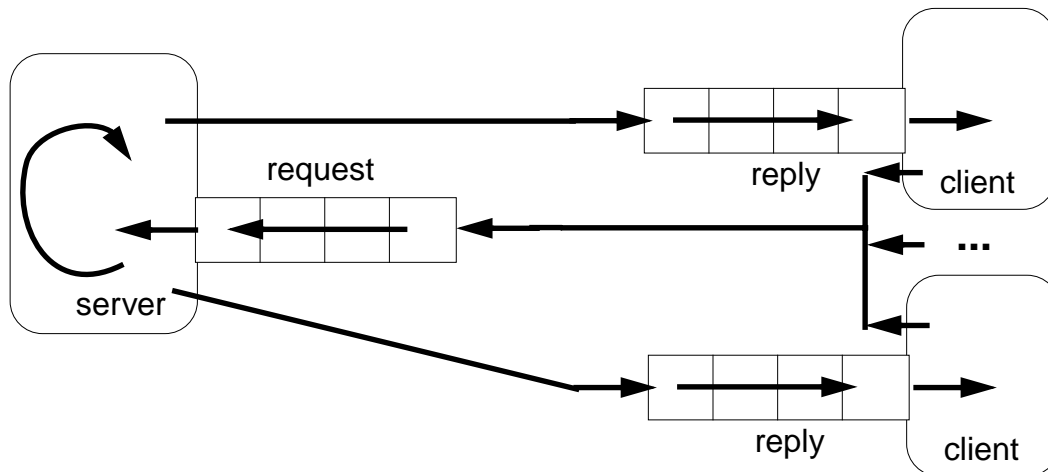
constructors

setKind (k), getKind ()

setArg (ind, val), getArg (ind), getArgList ()

# Client / server: basic channel structure

One **server process** responds to requests of several **client processes**



**request channel:**

input port of the server

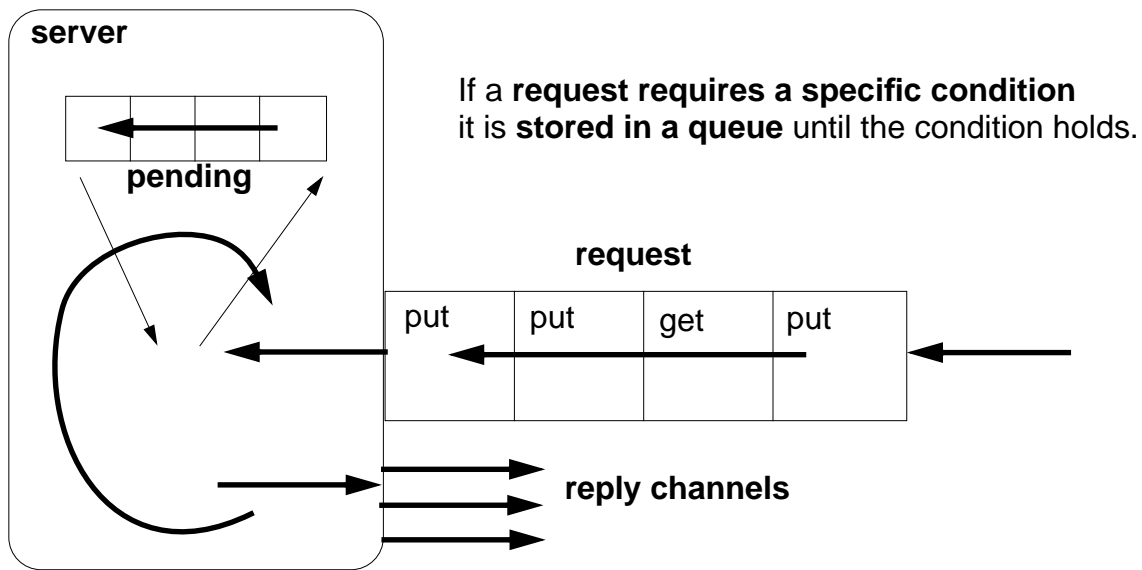
**reply channel:**

one for each client (input port),  
may be sent to the server included in the request message

**Application:** server distributes data or work packages on requests

## Server processes: different kinds of operations

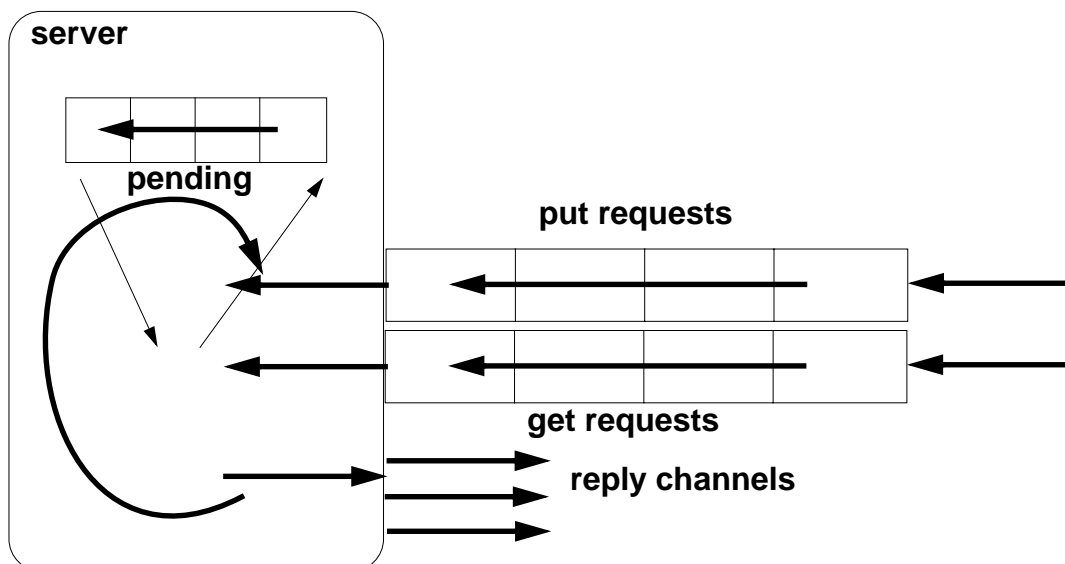
Different requests (operations) are represented by different kinds of messages.



The server processes the requests **strictly sequentially**; thus, it is guaranteed that critical sections are **not executed interleaved**.

**Termination:** terminate clients, empty channel, empty queue.

## Different kinds of operations on different channels

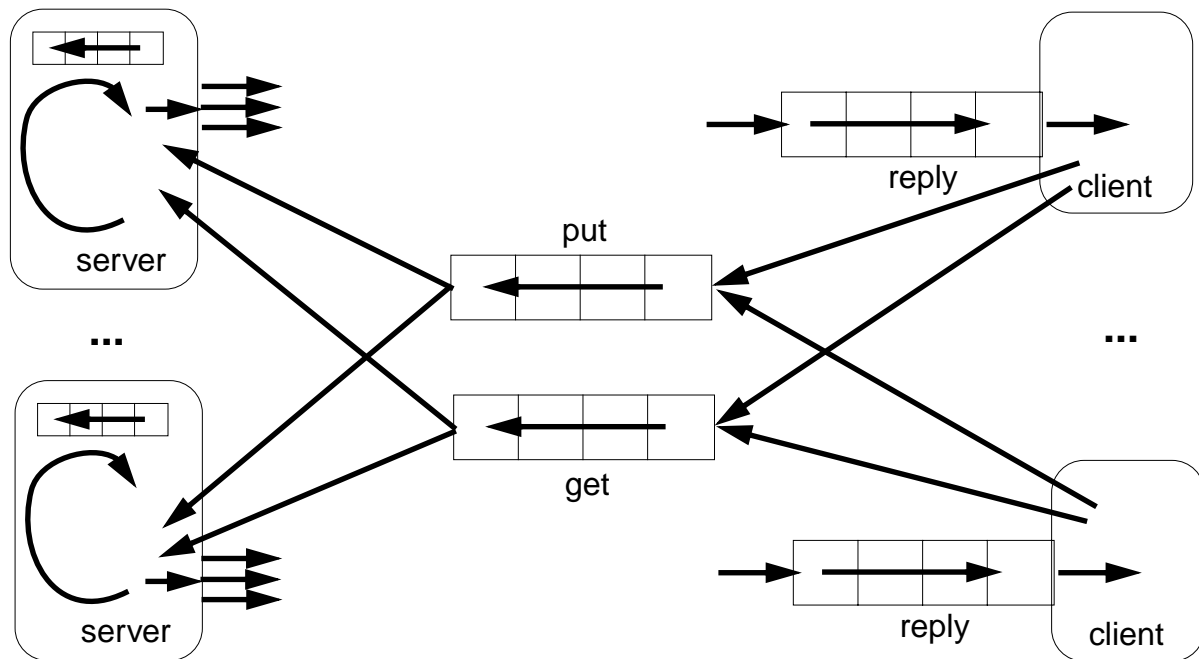


Server must not block on an empty input port while another port may be non-empty:

```
while (running) {
  if (!putPort.empty()) { msg = putPort.receive(); ... }
  if (!getPort.empty()) { msg = getPort.receive(); ... }
  if (!pending.empty()) { msg = pending.dequeue(); ... }
}
```

## Several servers

Several server processes, several client processes, several request channels



**Termination:** empty request channels, empty queues, empty reply channels

**Caution:** a receive on a channel may block a server!

## Receive without blocking

If several processes receive from a channel `ch`, then the check

```
if (!ch.empty()) msg = ch.receive();
```

may block.

That is not acceptable when several channels have to be checked in turn.

Hence, a new non-blocking channel method is introduced:

```
public class Channel
{
    ...
    public synchronized Object receiveMsgOrNull ()
    {
        if (msgQueue.empty()) return null;
        Object result = msgQueue.front();
        msgQueue.dequeue();
        return result;
    }
}
```

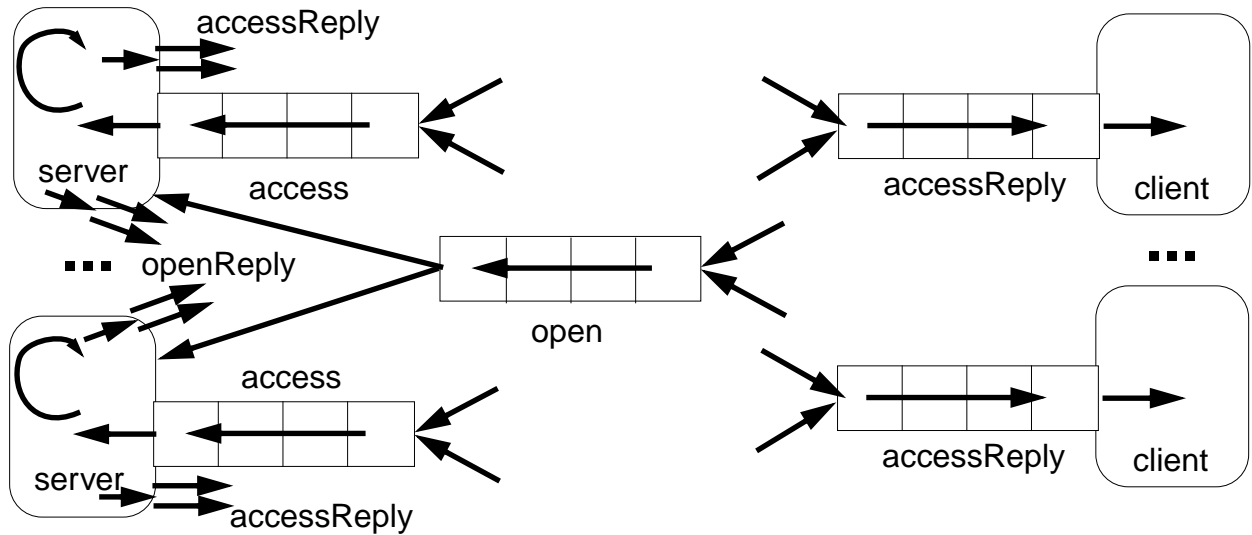
Checking several channels:

```
while (msg == null)
{
    if ((msg = ch1.receiveMsgOrNull()) == null)
        if ((msg = ch2.receiveMsgOrNull()) == null)
            Thread.sleep (500);
}
```

## Conversation sequences between client and server

Example for an **application pattern** is „file servers“:

- **several equivalent servers** respond to requests of **several clients**
- a client sends an **opening request** on a **channel common** for all servers (**open**)
- one server commits to the task; it then leads a conversation with the client according to a **specific protocol**, e. g. **(open openReply) ((read readReply) | (write writeReply))\* (close closeReply)**
- **reply channels** are contained in the **open** and **openReply** messages.



## Active monitor (server) vs. passive monitor

### active monitor

active process

request - reply via channels

kinds of messages and/or different channels

requests are handled sequentially

queue of pending requests  
replies are delayed

may cooperate on the same request channels

### 1. program structure

### 2. client communication

### 3. server operations

### 4. mutual exclusion

### 5. delayed service

### 6. multiple servers

### passive monitor

passive program module

calls of entry procedures

entry procedures

guaranteed for entry procedure calls

client processes are blocked  
condition variables, wait - signal

multiple monitors are not related



## 8. Messages in Distributed Systems

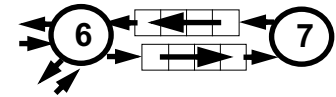
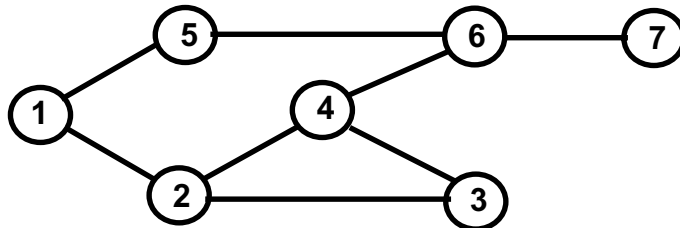
### Distributed processes: Broadcast in a net of processors

**Net:** bi-directional graph, connected, irregular structure;

**node:** a process

**edge:** a pair of links (channels) which connect two nodes in both directions

A node knows only its direct neighbours and the links to and from each neighbour:



#### Broadcast:

A message is sent from an initiator node such that it reaches every node in the net. Finally all channels have to be empty.

#### Problems:

- graph may have cycles
- nodes do not know the graph beyond their neighbours

## Broadcast method

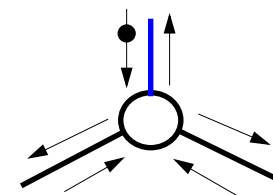
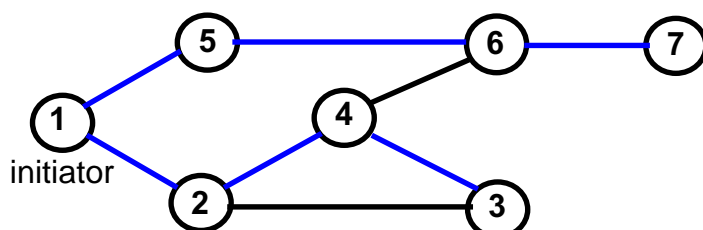
**Method** (for all nodes but the initiator node):

1. The node waits for a message on its incoming links.
2. After having **received the first message** it sends a **copy to all of its n neighbours** - including to the sender of the first message
3. The node then receives **n-1 redundant messages** from the remaining neighbours

All nodes are finally reached because of (2).

All channels are finally empty because of (3).

The connection to the sender of the first message is considered to be an edge of a **spanning tree** of the graph. That information may be used to simplify subsequent broadcasts.



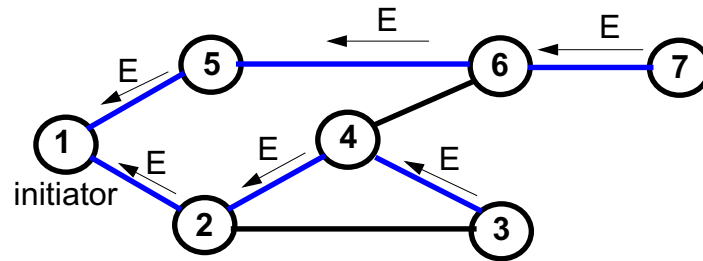
total number of messages:  $2 * |\text{edges}|$

## Probe and echo in a net

**Task:** An initiator requests combined **information from all nodes** in the graph (**probe**). The information is **combined** on its way through the net (**echo**);  
e. g. sum of certain values local to each node, topology of the graph, some global state.

### Method (roughly):

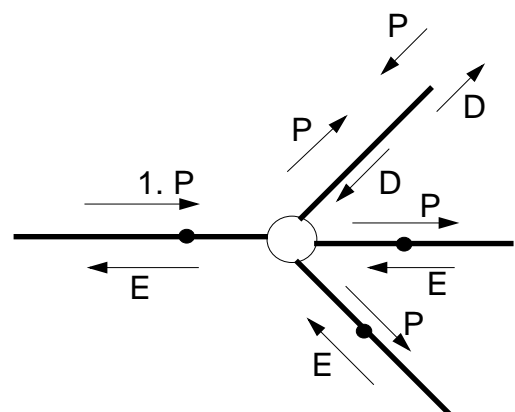
- distribute the probes like a broadcast,
- let the first reception determine a spanning tree,
- return the echoes on the spanning tree edges.



## Probe and echo: detailed operations

### Operations of each node (except the initiator):

- The node has **n neighbours** with an **incoming and outgoing link to each** of them.
- After having **received the first probe from neighbour s**, send a **probe to all neighbours except to s**, i. e. **n - 1 probes**.
- Each further **incoming probe** is replied with a **dummy** message.
- Wait until **n - 1 dummies and echoes** have arrived.
- Then combine the echoes and **send it to s**.



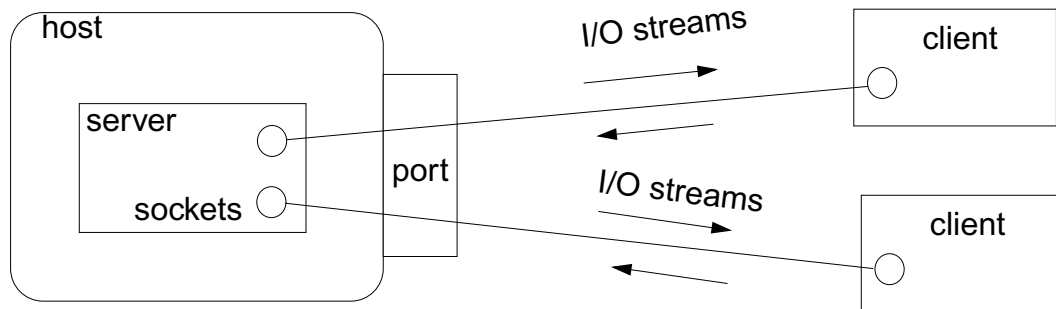
**2 messages** are sent on each **spanning tree edge**.

**4 messages** are sent on each **other edge**.

## Connections via ports and sockets

### Port:

- an **abstract connection point** of a computer; numerically encoded
- a **server process** is determined to **respond to a certain port**, e. g. port 13: date and time
- client processes on other machines may send requests via **machine name and port number**



### Socket:

- Abstraction of **network software** for communication via ports.
- Sockets are created from **machine address and port number**.
- **Several sockets** on one port may serve several clients.
- **I/O streams** can be setup on a socket.

## Sockets and I/O-streams

Get a machine address:

```
InetAddress  addr1 = InetAddress.getByName ("java.sun.com"),
              addr2 = InetAddress.getByName ("206.26.48.100"),
              addr3 = InetAddress.getLocalHost();
```

**Client side:** create a socket that connects to the server machine:

```
Socket myServer = new Socket (addr2, port);
```

Setup I/O-streams on the socket:

```
BufferedReader in =
    new BufferedReader
      (new InputStreamReader (myServer.getInputStream()));

PrintWriter out =
    new PrintWriter (myServer.getOutputStream(), true);
```

**Server side:** create a specific socket, accept incoming connections:

```
ServerSocket listener = new ServerSocket (port);
...
Socket client = listener.accept(); ... client.close();
```

## Worker paradigm

A task is decomposed dynamically in a **bag of subtasks**.  
 A set of **worker processes** of the same kind  
**solve subtasks** of the bag and may **create new ones**.

**Speedup** if the processes are executed  
 in parallel on different processors.

**Applications:** dynamically **decomposable** tasks, e.g.

- solving **combinatorial problems** with methods like Branch & Bound, Divide & Conquer, Backtracking
- image processing

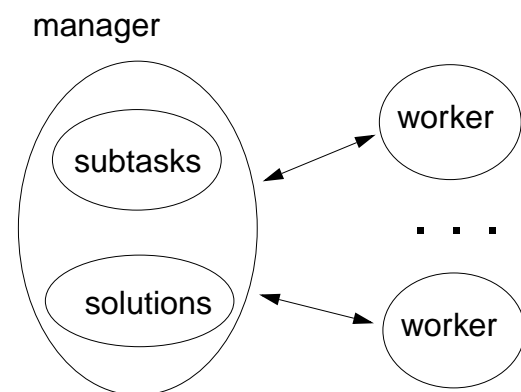
### general process structure:

#### manager process

manages the subtasks to be solved and  
 combines the solutions of the subtasks

#### worker process

solves one subtask after another,  
 creates new subtasks, and  
 provides solutions of subtasks.



## Branch and Bound

Algorithmic method for the solution of **combinatorial problems** (e. g. traveling salesperson)

**tree structured solution space** is searched for a best solution

### General scheme of operations:

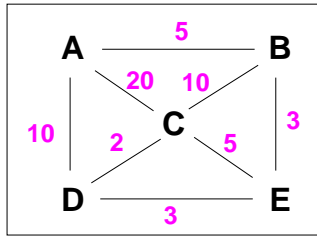
- **partial solution S is extended** to  $S_1, S_2, \dots$  (e. g. add an edge to a path)
- is a partial solution **valid**? (e. g. is the added node reached the first time?)
- is S a **complete** solution? (e. g. are all nodes reached)
- **MinCost (S) = C**: each solution that can be created from S has at least cost C (e. g. sum of the costs of the edges of S)
- **Bound**: costs of the best solution so far.

**Data structures:** a queue sorted according to MinCost; a bound variable

### sequential algorithm:

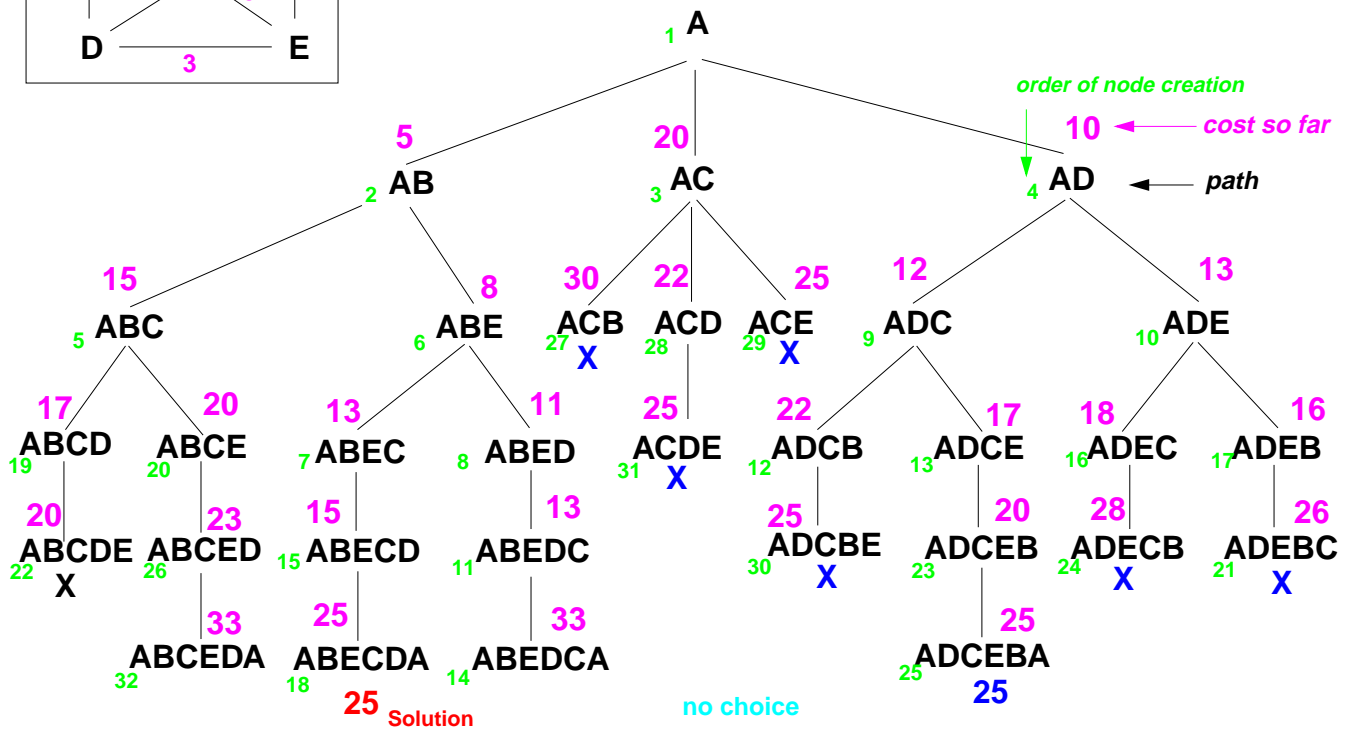
iterate until the queue is empty:  
 remove the first element and extend it  
 check the thus created new elements  
 a new solution and a better bound may be found  
 update the queue

# B&B example: Travelling sales person



Connection graph

## Solution space

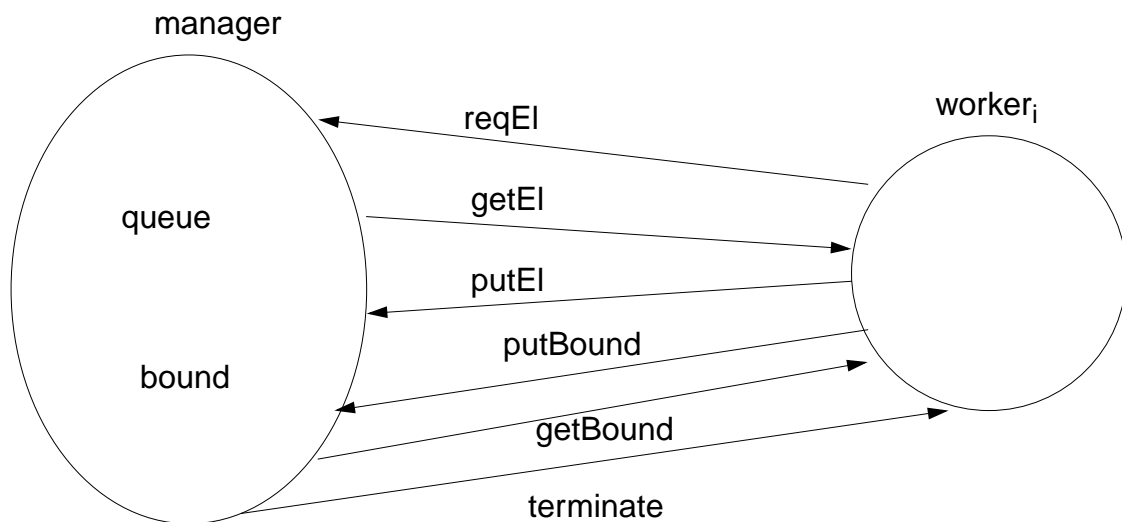


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# Parallel Branch & Bound (central)

A **central manager process** holds the queue and the bound variable

Each **worker process** extends an element, checks it, computes its costs, and a new bound



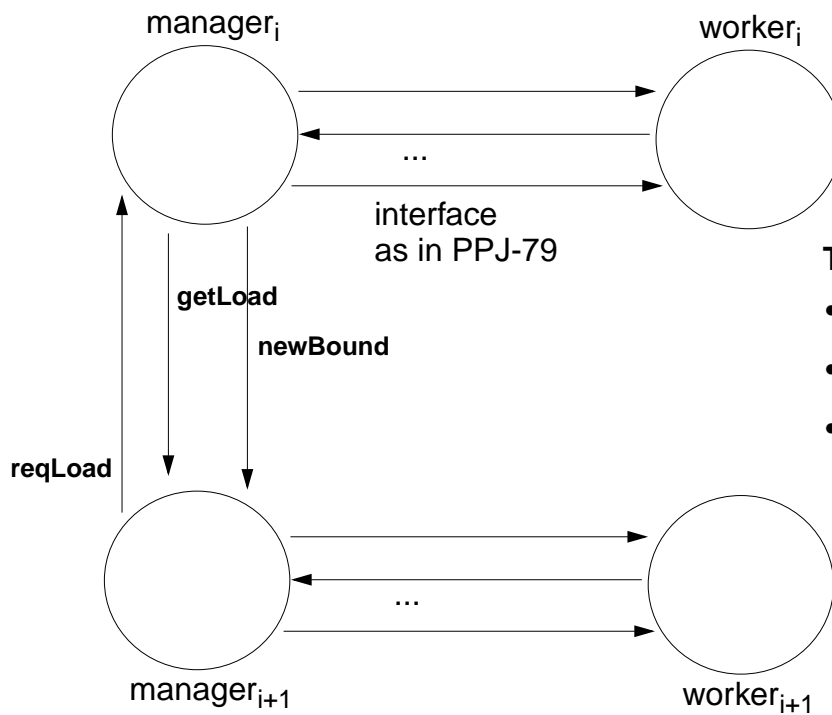
**Protocol:**  
for a single Worker  
 $reqEl (getEl [getBound] (putEl | putBound)^* reqEl)^* terminate$

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## Parallel Branch & Bound (distributed)

Several **manager processes cooperate** - one for each worker process.

The work load is balanced between neighbours, e. g. organized in a ring



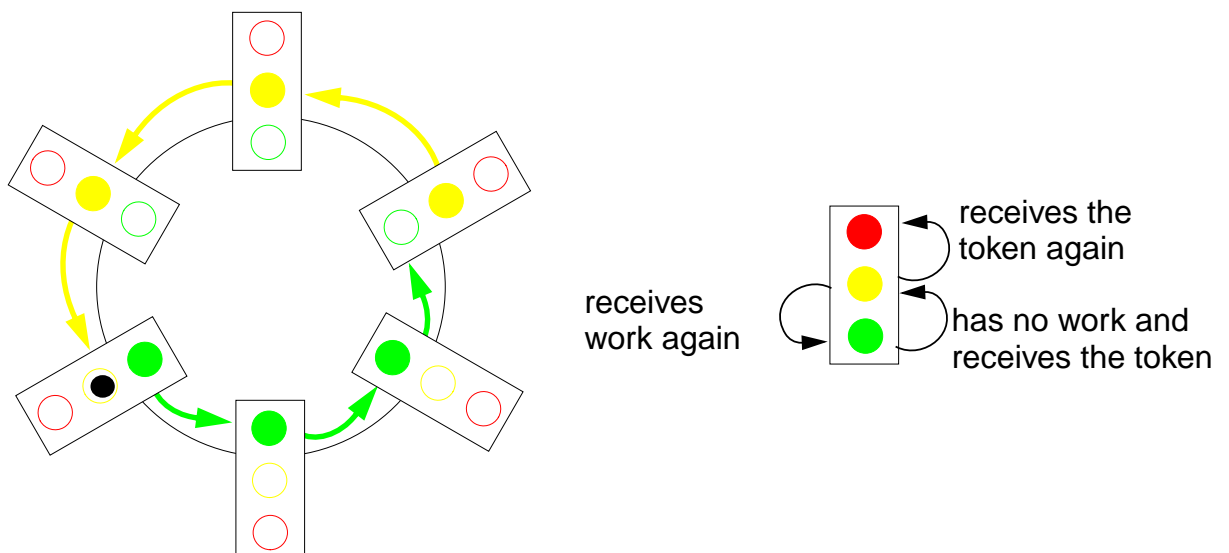
### Termination condition:

- all workers are inactive,
- no manager has another task
- all task channels are empty

## Termination in a ring

**Task:** Determine a **global state of processes** that communicate in a **directed ring**, and inform all processes, e. g. „all processes are inactive“.

**Idea:** A token rotates through the ring and marks the processes (**yellow**) that have reached the state in question (inactive). At the end of the marked sequence the mark may be reset again. When the token reaches the end of the marked sequence, the state holds globally

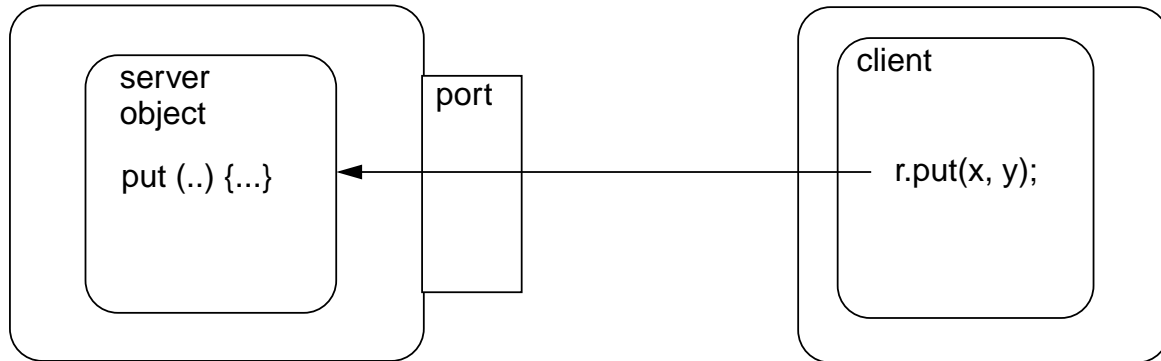


## Method calls for objects on remote machines (RMI)

**Remote Method Invocation (RMI):** Call of a method for an object that is on a remote machine

In Java RMI is available via the library `java.rmi`.

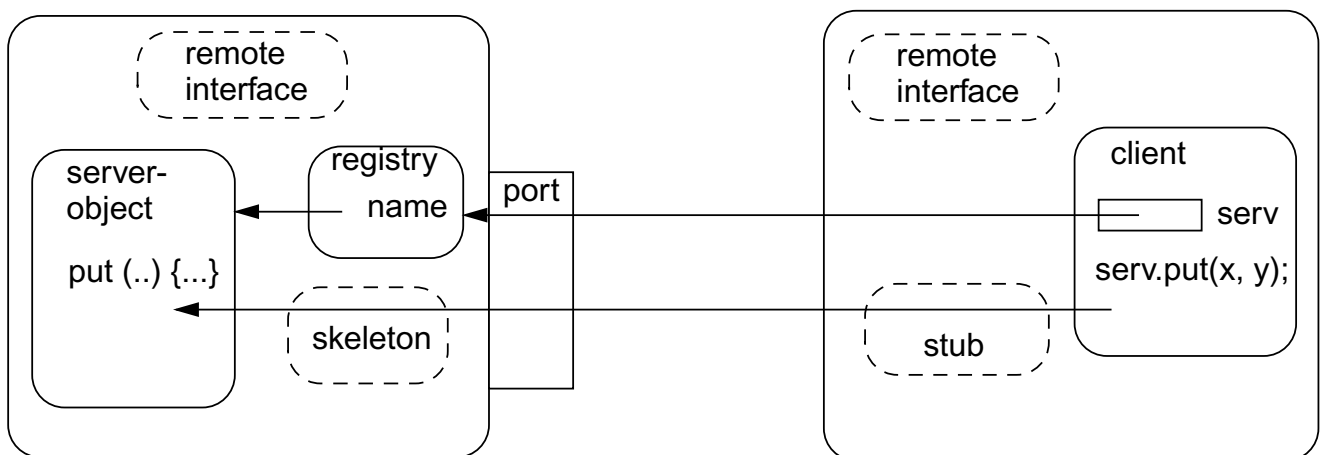
**Comparable techniques:** CORBA with IDL, Microsoft DCOM with COM



### Tasks:

- **identify objects** across machine borders (object management, naming service)
- **interface** for remote accesses and executable proxies for the remote objects (skeleton, stub)
- **method call**, parameter and result are transferred (object serialization)

## RMI in Java



**remote interface:** special requirements for interface methods

**registry:** system process for the machine and for a port;  
establishes relations between names and object references

**server skeleton:** proxy of the server for remote accesses to server objects,  
performs I/O transfer on the server side,

**client stub:** proxy of the server, performs I/O transfer on the client side

## RMI development steps

Example: make a `Hashtable` available as a server object

1. Define a remote interface:

```
public interface RemoteMap extends java.rmi.Remote
{ public Object get (Object key) throws RemoteException; ...}
```

2. Develop an adapter class to adapt the server class to a remote interface:

```
public class RemoteMapAdapter extends UnicastRemoteObject
    implements RemoteMap
{ public RemoteMapAdapter (Hashtable a) { adaptee = a; }
  public Object get (Object key) throws RemoteException
  { return adaptee.get (key); }
  ...
}
```

3. Server main program creates the server object and enters it into the registry:

```
Hashtable adaptee = new Hashtable();
RemoteMapAdapter adapter = new RemoteMapAdapter (adaptee);
Naming.rebind (registeredObjectName, adapter);
```

4. Generate the skeleton and stub from the adapted server class; copy the client stub on to the client machine:

```
rmic RemoteMapAdapter
```

## RMI development steps (continued)

5. Client identifies the server object on a target machine and calls methods:

```
Registry remoteRegistry = LocateRegistry.getRegistry (hostName);
RemoteMap serv = (RemoteMap) remoteRegistry.lookup (remObjectName);
v = serv.get (key);
```

6. Start a registry on the server machine:

```
rmiregistry [port] &
Default Port is 1099
```

7. Start some servers on the server machine.

8. Start some clients on client machines.



## Objects as parameters of RMI calls

**Parameters and results of RMI calls** are transferred via I/O streams.

That is straight-forward for values of **basic types** and **strings**.

### For objects in general:

The values of their variables are transferred,  
on the receiver side a new object is created from those values.

The class of such objects has to implement the interface **Serializable**:

```
import java.io.Serializable;

class SIPair implements java.io.Serializable
{   private String s;
    private int i;

    public SIPair (String a, int b) { s = a; i = b; }
    public String toString () { return s + "-" + i; }
}
```

## 9. Synchronous message passing

**Processes communicate and synchronize directly**,  
space is provided for **only one message** (instead of a channel).

### Operations:

- **send (b)**: **blocks** until the partner process is ready to receive the message
- **receive (v)**: blocks until the partner process is ready to send a message.

When both sender and receiver processes are ready for the communication,  
the message is transferred, like an assignment  $v := b$ ;

A send-receive-pair is both **data transfer and synchronization point**

**Origin:** Communicating Sequential Processes (CSP) [C.A.R. Hoare, CACM 21, 8, 1978]



## Notations for synchronous message passing

**Notation** in CSP und Occam:

**p:** ... **q ! ex ...** **send** the value of the expression **ex** to process **q**

**q:** ... **p ? v ...** **receive** a value from process **p** and assign it to variable **v**

**multiple ports** and **composed messages** may be used:

**p:** ... **q ! Port1 (a1,...,an) ...**

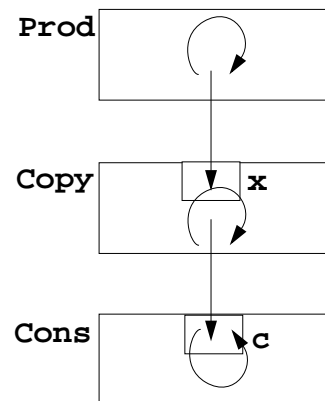
**q:** ... **p ? Port1 (v1,...,vn) ...**

**Example:** copy data from a producer to a consumer:

```
Prod:  var p: int;
       do true -> p :=...; Copy ! p od

Copy:  var x: int;
       do true -> Prod ? x; Cons ! x od

Cons:  var c: int;
       do true -> Copy ? c; ... od
```



## Selective wait

**Guarded command:** (invented by E. W. Dijkstra)

a branch may be taken, if a **condition** is true and a **communication** is enabled (**guard**)

```
if Condition1; p ! x -> Statement1
[] Condition2; q ? y -> Statement2
[] Condition3; r ? z -> Statement3
fi
```

**A communication statement in a guard yields**

- true**, if the partner process is ready to communicate
- false**, if the partner process is terminated,
- open** otherwise (process is not ready, not terminated)

**Execution of a guarded command** depends on the guards:

- If **some guards are true**, one of them is chosen, the communication and the branch statement are executed.
- If **all guards are false** the guarded command is completed without executing anything.
- **Otherwise** the process is blocked until one of the above cases holds.

Notation of an **indexed selection**:

```
if (i: 1..n) Condition; p[i] ? v -> Statements fi
```

## Guarded loops

A **guarded loop** repeats the execution of its guarded command **until all guards yield false**:

```
do
  Condition1; p ! x-> Statement1
[] Condition2; r ? z-> Statement2
od
```

**Example:** bounded buffer:

```
process Buffer
  do
    cnt < N; Prod ? buf[rear] -> cnt++; rear := rear % N + 1;
  [] cnt > 0; Cons ! buf[front] -> cnt--; front := front % N + 1;
  od
end

process Prod
  var p:=0: int;
  do p<42; Buffer ! p -> p:=p+1;
  od
end

process Cons
  var c: int;
  do Buffer ? c -> print c;
  od
end
```

## Prefix sums computed with synchronous messages

Synchronous communication provides both **transfer of data and synchronization**.

**Necessary synchronization only** (cf. synchronous barriers, PPJ-48)

```
const N := 6; var a [0:N-1] : int;

process Worker (i := 0 to N-1)           a process for each element
  var d := 1, sum, new: int

  sum := a[i];

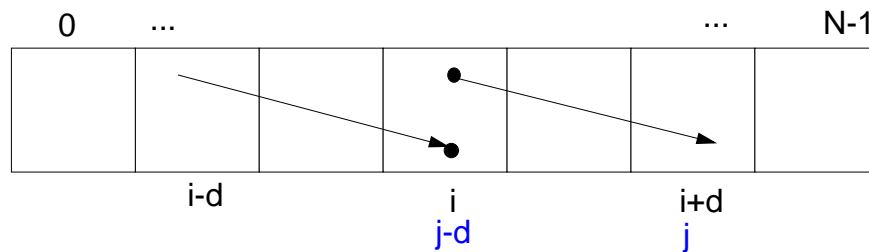
  {Invariant SUM: sum = a[i-d+1] + ... + a[i]}

  do d < N-1 ->
    if (i+d) < N -> Worker(i+d) ! sum fi      shift old value to the right
    if (i-d) >= 0 -> Worker(i-d) ? new; sum := sum + new fi
                                                get new value from the left
    d := 2*d                                  double the distance
  od
end                                         {SUM and d >= N-1}
```

Why can deadlocks not occur?

## No deadlocks in synchronous prefix sums

synchronization pattern



- **! and ?** operations occur always **in pairs**:

if  $i+d < N$  and  $i \geq 0$  process  $i$  executes `Worker(i+d)!sum`  
 let  $j = i+d$ , i.e.  $j-d = i \geq 0$ , hence process  $j$  executes `Worker(j-d)?new`

- There is always a process that does **not send but receives**:

Choose  $i$  such that  $i < N$  and  $i+d \geq N$ , then process  $i$  only receives:  
 Prove by induction.

- As **no process first receives and then sends**, there is **no deadlock**

## Client/Server scheme with synchronous messages

### Technique:

for each **kind of operation** that the server offers, a communication via **2 ports**:

- `oprReq` for transfer of the parameters
- `oprRepl` for transfer of the reply

Scheme of the **client processes**:

```
process Client (I := 1 to N)
  ...
  Server ! oprReq (myArgs)
  Server ? oprRepl (myRes)
  ...
end
```

Scheme of the **server process**:

```
process Server ()
  ...
  do (c: 1..N) ConditionOpr1; Client[c] ? oprReq(oprArgs)
    -> process the request ...
    Client[c] ! oprRepl(oprResults)
  [] correspondingly for other operations ...
  od
end
```

## Synchronous Client/Server: variants and comparison

Synchronous servers have the  
**same characteristics as asynchronous servers**,  
 i. e. active monitors (PPJ-70).

### Variants of synchronous servers:

1. Extension to **multiple instances of servers**:  
 use **guarded command loops** to check  
 whether a communication is enabled
2. If an operation can **not be executed immediately**,  
 it has to be delayed, and  
 its arguments have to be stored in a pending queue
3. The **reply port can be omitted** if
  - there is no result returned, and
  - the request is never delayed
4. Special case: resource allocation with request and release.
5. **Conversation sequences** are executed in the part „process the request“.  
**Conversation protocols** are implemented by a  
 sequence of send, receive, and guarded commands.

## Synchronous messages in Occam

### Occam:

- concurrent programming language, based on **CSP**
- initially developed in 1983 at INMOS Ltd. as native language for **INMOS Transputer** systems
- a program is a nested structure of parallel processes (**PAR**), sequential code blocks (**SEQ**), guarded commands (**ALT**), synchronous send (!) and receive (?) operations, procedures, imperative statement forms;
- communication via **1:1 channels**
- fundamental data types, arrays, records
- extended 2006 to **Occam-pi**, University of Kent, GB  
**pi-calculus** (Milner et. al, 1999):  
 formal process calculus where names of channels can be communicated via channels  
 Kent Retargetable occam Compiler (**KRoC**)  
 (open source)

```

CHAN OF INT chn:
PAR
  SEQ
    INT a:
    a := 42
    chn ! a

  SEQ
    INT b:
    chn ? b
    b := b + 1
  
```

## Bounded Buffer in Occam

```

CHAN OF Data in, out:
  PAR
    SEQ -- process buffer
      Queue (k) buf:
      Data d:
      WHILE TRUE
        ALT
          in ? d & length(buf) < k
            enqueue(buf, d)
          out ! front(buf) & length(buf) > 0
            ! not allowed in a guard
            dequeue(buf)

```

```

SEQ
  -- only one producer process
  Data d:
  WHILE TRUE
    SEQ
      d = produce ()
      in ! d

```

```

SEQ
  -- only one consumer process
  Data d:
  WHILE TRUE
    SEQ
      out ? d
      consume (d)

```

## Synchronous rendezvous in Ada

### Ada:

- **general purpose** programming language dedicated for **embedded systems**
- 1979: Jean Ichbiah at CII-Honeywell-Bull (Paris) wins a **competition** of language proposals initiated by the **US DoD**
- **Ada 83 reference manual**
- **Ada 95 ISO Standard**, including oo constructs
- **Ada 2005**, extensions
- **concurrency notions:** processes (**task**, **task type**), shared data, synchronous communication (**rendezvous**), entry operations pass data in both directions, guarded commands (**select**, **accept**)

```

task type Producer;

task body Producer is
  d: Data;
begin
  loop
    d := produce ();
    Buffer.Put (d);
  end loop;
end Producer;

task type Consumer;

task body Consumer is
  d: Data;
begin
  loop
    Buffer.Get (d);
    consume (d);
  end loop;
end Consumer;

```

## Ada: Synchronous rendezvous

```

task type Buffer is      -- interface
  entry Put (d: in Data); -- input port
  entry Get (d: out Data); -- output port
end Buffer;

task body Buffer is
  buf: Queue (k);
  d: Data;
begin
  loop
    select              -- guarded command
      when length(buf) < k =>
        accept Put (d: in Data) do
          enqueue(buf, d);
        end Put;
      or
      when length(buf) > 0 =>
        accept Get (d: out Data) do
          d := front(buf);
          dequeue(buf);
        end Get;
    end select;
  end loop;
end Buffer;

```

```

task type Producer;

task body Producer is
  d: Data;
begin
  loop
    d := produce ();
    Buffer.Put (d);
  end loop;
end Producer;

task type Consumer;

task body Consumer is
  d: Data;
begin
  loop
    Buffer.Get (d);
    consume (d);
  end loop;
end Consumer;

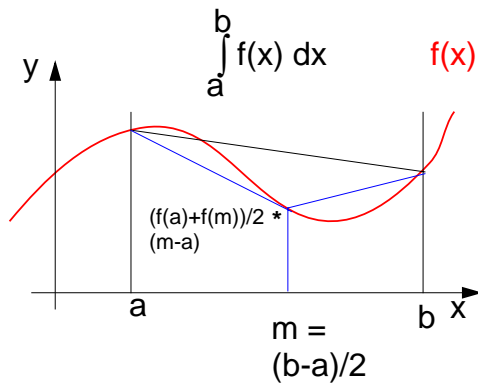
```

## 10. Concurrent and functional programming

### Overview

1. **Pure functional programs do not have side-effects:**  
operands of an operation and arguments of a call  
can be **evaluated in any order**, in particular **concurrently**
2. **Recursive task decomposition** can be parallelized according to the  
paradigm **bag of subtasks**
3. **Lazy evaluation** of lists leads to **programs that transform streams**, can be  
**parallelized** according the **pipelining** paradigma
4. **Dataflow languages** and dataflow machines support **stream programming**
5. **Concurrency notions in functional languages:**  
**Message passing in Erlang**  
**Actors in Scala**

## Recursive adaptive quadrature computation



```

fun quad (f, l, r, area, eps) =
  let m = (r-l)/2 and
      fl = f(l) and
      fm = f(m) and
      fr = f(r) and
      larea = (fl+fm)*(m-l)/2 and
      rarea = (fm+fr)*(r-m)/2 and
  in
    if abs(larea+rarea-area)>eps
    then
      let

```

Compute an **approximation of the integral** over  $f(x)$  between  $a$  and  $b$ .

Recursively **refine the interval** into two subintervals until the sum of the **areas of the two trapezoids** differs less than  $\text{eps}$  from the **area of the big trapezoid**.

See [G. Andrews: Foundations of Multithreaded, Parallel, and Distributed Programming, Addison Wesley, 2000, pp. 17-19]

```

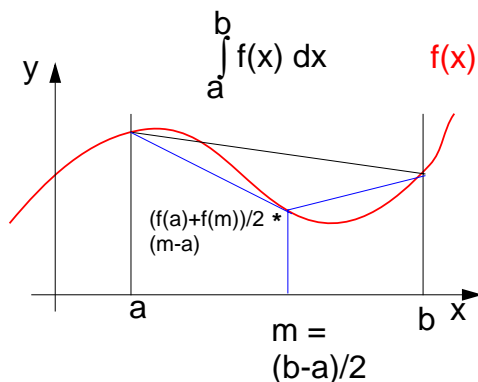
      let
        lar = quad(f,l,m,larea,eps) and
        rar = quad(f,m,r,rarea,eps)
      in (lar+rar)
    end
  else area
end

```

initial call:

```
quad (f,a,b,(f(a)+f(b))/2*(b-a),0.001)
```

## Recursive adaptive quadrature computation



```

fun quad (f, l, r, area, eps) =
  let m = (r-l)/2 and
      fl = f(l) and
      fm = f(m) and
      fr = f(r) and
      larea = (fl+fm)*(m-l)/2 and
      rarea = (fm+fr)*(r-m)/2 and
  in
    if abs(larea+rarea-area)>eps
    then
      let

```

Compute an **approximation of the integral** over  $f(x)$  between  $a$  and  $b$ .

Recursively **refine the interval** into two subintervals until the sum of the **areas of the two trapezoids** differs less than  $\text{eps}$  from the **area of the big trapezoid**.

**Fork two concurrent processes.**

See [G. Andrews: Foundations of Multithreaded, Parallel, and Distributed Programming, Addison Wesley, 2000, pp. 17-19]

```

      let
        co
          lar = quad(f,l,m,larea,eps) and
        //
          rar = quad(f,m,r,rarea,eps)
        oc
      in (lar+rar)
    end
  else area
end

```

initial call:

```
quad (f,a,b,(f(a)+f(b))/2*(b-a),0.001)
```



## Streams in functional programming

**Linear lists** are fundamental data structures in functional programming, e.g. in **SML**:

```
datatype 'a list = nil | :: of 'a * 'a list
```

**Eager evaluation:** all elements of a list are to be computed, before any can be accessed.

**Lazy evaluation** only those elements of a list are computed which are going to be accessed.

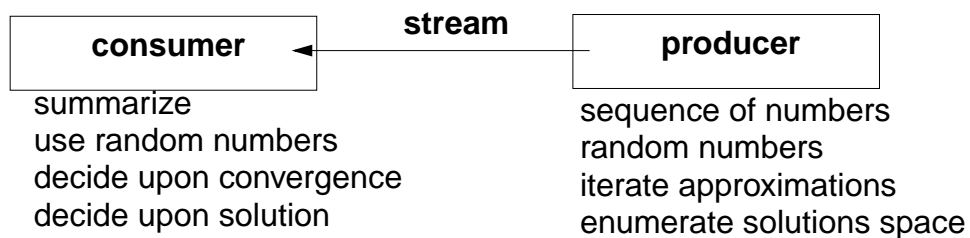
That can be achieved by replacing the (pointer to) the tail of the list by a parameterless function which computes the tail of the sequence when needed:

```
datatype 'a seq = Nil | Cons of 'a * (unit -> 'a seq)
```

Lazy lists are called **streams**.

Streams establish a useful **programming paradigm**:

Programming the **creation** of a stream can be **separated** from programming its **use**.



**Functions on streams can be understood as communicating concurrent processes.**

## Examples for stream functions (1)

produce a stream of numbers:

```
int -> int seq
fun from k = Cons (k, fn()=> from (k+1));
```

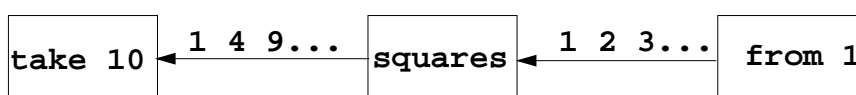
consume the first n elements into a list:

```
'a seq * int -> 'a list
fun take (xq, 0) = []
  | take (Nil, n) = raise Empty
  | take (Cons(x, xf), n) = x :: take (xf (), n - 1);
```

transform a stream of numbers:

```
int seq -> int seq
fun squares Nil = Nil
  | squares (Cons (x, xf)) = Cons (x * x, fn() => squares (xf()));
```

```
take (squares (from 1), 10);
```



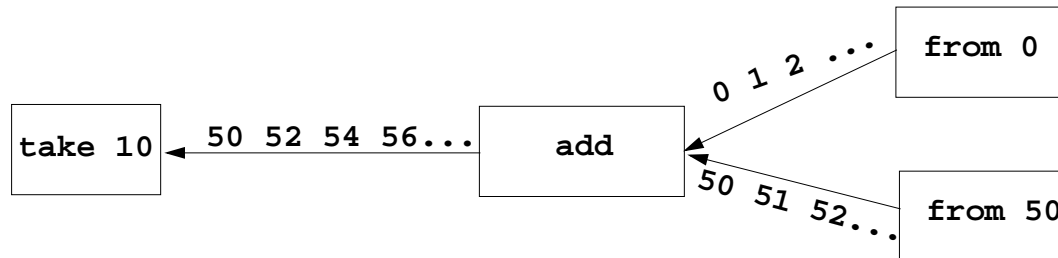
## Examples for stream functions (2)

add the numbers of two streams: `int seq * int seq -> int seq`

```

fun add (Cons(x, xf), Cons(y, yf)) =
  Cons (x+y, fn() => add (xf(), yf()))
|   add _ = Nil;

```



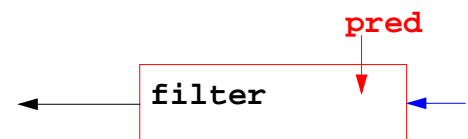
### Filter-Schema:

`('a -> bool) -> 'a seq -> 'a seq`

```

fun filter pred Nil = Nil
|   filter pred (Cons(x,xf)) =
  if pred x then Cons (x, fn()=> filter pred (xf()))
  else filter pred (xf());

```



## Recursive stream composition

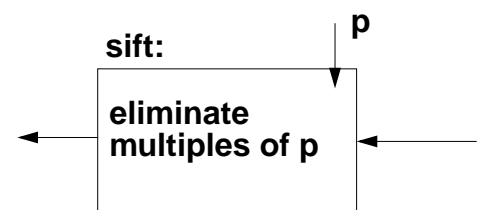
```

fun sift p =
  filter (fn n => n mod p <> 0);

fun sieve (Cons(p,nf)) =
  Cons (p, fn() => sieve (sift p (nf())));

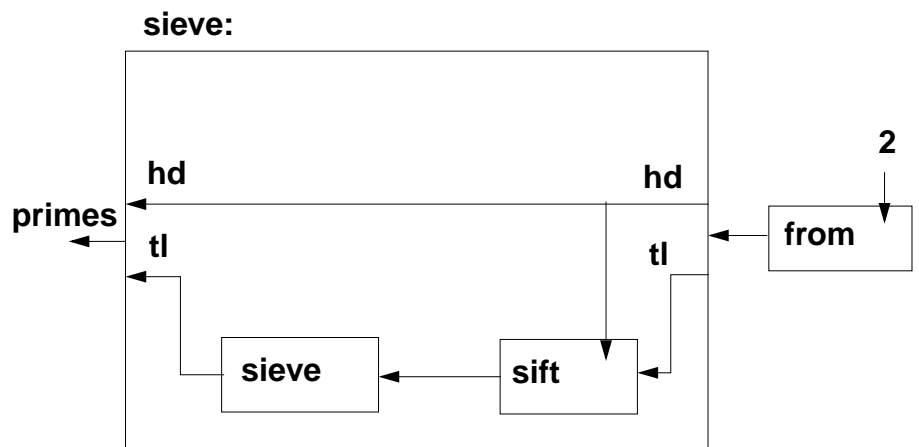
val primes = sieve (from 2);
take (primes, 25);

```



Compute prime numbers:

Sieve of Eratosthenes



All recursively constructed sift-sieve-pairs can execute concurrently!

## Sieve of Eratosthenes in CSP

A pipeline of filters:

**L processes** are created, each sends a **stream of numbers** to its successor.

The **first number p** received is a prime. It is **used to filter** the following numbers.

Finally, each process holds a prime in p.

```

process Sieve[1]
  for [1 = 3 to n by 2]
    Sieve[2] ! i # pass odd numbers to Sieve[2]

process Sieve[i = 2 to L]
  int p, next
  Sieve[i-1] ? p          # p is a prime
  do Sieve[i-1] ? next -> # receive next candidate
    if (next mod p) != 0 ->
      Sieve[i+1] ! next # pass it on
    fi
  od
  
```

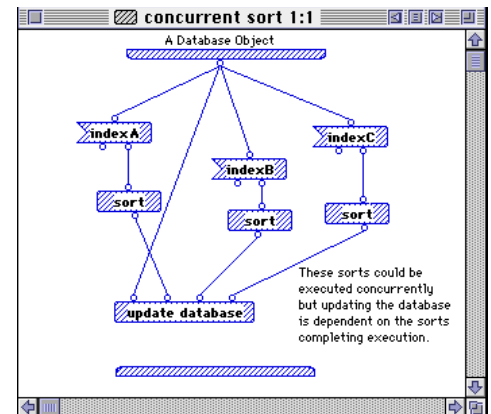
[G. Andrews: Foundations of Multithreaded, Parallel, and Distributed Programming, Addison Wesley, 2000, pp. 326-328]

## Dataflow languages

Textual languages:

**Lucid**: stream computations by equations, no side effects; 1976, Wadge, Ashcroft

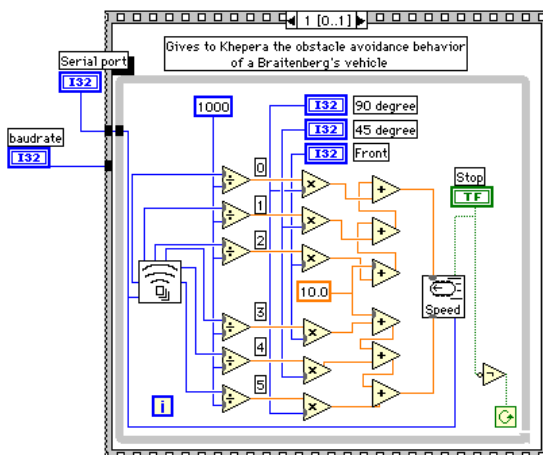
**SISAL**: (Streams and Iteration in a Single Assignment Language), no side-effects, fine-grained parallelization by compiler, 1983



Visual languages:

**Prograph (Acadia University 1983)**: dataflow and object-oriented

**LabVIEW (National Instruments, 1986)**: Nodes represent stream processing functions connected by wires, concurrent execution triggered by available input. Strong support of interfaces to instrumentation hardware.



# Language Erlang

**Erlang** developed 1986 by Joe Armstrong, et.al at **Ericsson**

- multi-paradigm: **functional** and **concurrent**
- initial application area: **telecommunication**  
requirements: distributed, fault-tolerant, soft-real-time, non-stopping software
- **processes** communicate via **asynchronous message** passing
- **single-assignment** variables, **no shared memory** between processes

Explanations and examples taken from

[J. Armstrong, R. Virding, C. Wikström, M. Williams: Concurrent Programming in ERLANG, Second Edition, Ericsson Telecommunications Systems Laboratories, Prentice Hall, 1996]

<http://www.erlang.org>

## Basic communication constructs

**process creation:**

```
Pid = spawn(Module, FunctionName, ArgumentList)
```

**asynchronous message send:**

```
Pid ! Message
```

The operands are expressions which yield a process id and a message.

**selective receive:**

```
receive
  Pattern1 [when Guard1] ->
    Actions1 ;
  Pattern2 [when Guard2] ->
    Actions2 ;
  ...
end
```

Searches the process' **mailbox** for a **message that matches a pattern**, and receives it.  
**Can not block on an unexpected message!**

### Initial example

A module that creates counter processes:

```
-module(counter).
-export([start/0,loop/1]).

start() ->
  spawn(counter, loop, [0]).

loop(Val) ->
  receive
    increment ->
      loop(Val + 1)
  end.
```

clients send **increment** messages to it

## Complete example: Counter

**Interface functions** are called by client processes.

They send 3 kinds of messages.

`self()` gives the client's pid, to reply to it.

The counter process identifies itself in the reply.

The receive is iterated (tail-recursion).

Unexpected messages are removed

```
-module(counter).
-export([start/0,loop/1,increment/1,value/1,stop/1]).

%% First the interface functions.
start() -> spawn(counter, loop, [0]).

increment(Counter) -> Counter ! increment.

value(Counter) ->
    Counter ! {self(),value},
    receive {Counter,Value} -> Value
end.

stop(Counter) -> Counter ! stop.

%% The counter loop.
loop(Val) ->
    receive increment -> loop(Val + 1);
             {From,value} -> From ! {self(),Val},
             loop(Val);

             stop -> true;
             Other -> loop(Val)
end.
```

## Example: Allocation server (interface)

A server maintains two lists of **free and allocated resources**. Clients call a function `allocate` to request a resource and a function `free` to return that resource.

The two lists of **free and allocated resources** are initialized.

`register` associates the pid to a name.

The calls of `allocate` and `free` are transformed into **different kinds of messages**. Thus, implementation details are not disclosed to clients.

```
-module(allocator).
-export([start/1,server/2,allocate/0,free/1]).

start(Resources) ->
    Pid = spawn(allocator, server,
                [Resources,[]]),
    register(resource_alloc, Pid).

% The interface functions.

allocate() -> request(alloc).

free(Resource) -> request({free,Resource}).

request(Request) ->
    resource_alloc ! {self(),Request},
    receive {resource_alloc,Reply} -> Reply
end.
```

## Example: Allocation server (implementation)

The function `server` receives the two kinds of messages and transforms them into calls of `s_allocate` and `s_free`.

`s_allocate` returns **yes** and the resource or **no**, and **updates** the two lists in the recursive `server` call.

`s_free`: `member` checks whether the returned resource `R` is in the free list, returns `ok` and updates the lists,

or it returns `error`.

The `server` call loops.

```
server(Free, Allocated) ->
  receive
    {From, alloc} ->
      s_allocate(Free, Allocated, From);
    {From, {free, R}} ->
      s_free(Free, Allocated, From, R)
  end.

s_allocate([R|Free], Allocated, From) ->
  From ! {resource_alloc, {yes, R}},
  server(Free, [{R, From}|Allocated]);
s_allocate([], Allocated, From) ->
  From ! {resource_alloc, no},
  server([], Allocated).

s_free(Free, Allocated, From, R) ->
  case member({R, From}, Allocated) of
    true -> From ! {resource_alloc, ok},
      server([R|Free],
              delete({R, From},
                    Allocated));
    false -> From ! {resource_alloc, error},
      server(Free, Allocated)
  end.
```

## Scala: object-oriented and functional language

**Scala**: Object-oriented language (like Java, more compact notation), augmented by functional constructs (as in SML); object-oriented execution model (Java)

### functional constructs:

- nested functions, higher order functions, currying, case constructs based on pattern matching
- functions on lists, streams, ... provided in a big language library
- parametric polymorphism; restricted local type inference

### object-oriented constructs:

- classes define all types (types are consequently oo - including basic types), subtyping, restrictable type parameters, case classes
- object-oriented mixins (traits)

### general:

- static typing, parametric polymorphism and subtyping polymorphism
- very compact functional notation
- complex language, and quite complex language description
- compilable and executable together with Java classes
- since 2003, author: Martin Odersky, [www.scala.org](http://www.scala.org), [docs.scala-lang.org](http://docs.scala-lang.org)

## Actors in Scala (1)

An **actor** is a lightweight process:

- **actor** { **body** } creates a process that executes **body**
- **asynchronous** message passing
- **send**: **p ! msg** puts **msg** into **p**'s mailbox
- **receive** operation searches the mailbox for the first message that matches one of the case patterns (as in **Erlang**)
- **case x** is a catch-all pattern

Example: orders and cancellations

```
val orderMngr = actor {
  while (true)
    receive {
      case Order(sender, item) =>
        val o =
          handleOrder(sender, item)
        sender ! Ack(o)
      case Cancel(sender, o) =>
        if (o.pending) {
          cancelOrder(o)
          sender ! Ack(o)
        } else sender ! NoAck
      case x => junk += x
    }
}

val customer = actor {
  orderMngr ! Order(self, myItem)
  receive {
    case Ack(o) => ...
  }
}
```

[P. Haller, M. Odersky: Actors That Unify Threads and Events; in A.L. Murphy and J. Vitek (Eds.): COORDINATION 2007, LNCS 4467, pp. 171–190, 2007. © Springer-Verlag Berlin Heidelberg 2007]

## Actors in Scala (2)

Constructs used to simplify replying:

- The sender of a received message is stored in **sender**
- **reply(msg)** sends **msg** to **sender**
- **a !? msg** sends **msg** to **a**, waits for a reply, and returns it.

Example: orders and cancellations

```
val orderMngr = actor {
  while (true)
    receive {
      case Order(item) =>
        val o =
          handleOrder(sender, item)
        reply(Ack(o))
      case Cancel(o) =>
        if (o.pending) {
          cancelOrder(o)
          reply(Ack(o))
        } else reply(NoAck)
      case x => junk += x
    }
}

val customer = actor {
  orderMngr !? Order(myItem)
  match {
    case Ack(o) => ...
  }
}
```

[P. Haller, M. Odersky: Actors That Unify Threads and Events; in A.L. Murphy and J. Vitek (Eds.): COORDINATION 2007, LNCS 4467, pp. 171–190, 2007. © Springer-Verlag Berlin Heidelberg 2007]

## 11. Check your knowledge (1)

### Introduction

1. Explain the notions: sequential, parallel, interleaved, concurrent execution of processes.
2. How are Threads created in Java (3 steps)?

### Properties of Parallel Programs

3. Explain axioms and inference rules in Hoare Logic.
4. What does the weakest precondition  $wp(s, Q) = P$  mean?
5. Explain the notions: atomic action, at-most-once property.
6. How is interference between processes defined?
7. How is non-interference between processes proven?
8. Explain techniques to avoid interference between processes.

### Monitors

9. Explain how the two kinds of synchronization are used in monitors.
10. Explain the semantics of condition variables and the variants thereof.
11. Which are the 3 reasons why a process may wait for a monitor?
12. How do you implement several conditions with a single condition variable?

## Check your knowledge (2)

13. Signal-and-continue requires loops to check waiting-conditions. Why?
14. Explain the properties of monitors in Java.
15. When can notify be used instead of notifyAll?
16. Where does a monitor invariant hold? Where has it to be proven?
17. Explain how monitors are systematically developed in 5 steps.
18. Formulate a monitor invariant for the readers/writers scheme?
19. Explain the development steps for the method „Rendezvous of processes“.
20. How are waiting conditions and release operations inserted when using the method of counting variables?

### Barriers

21. Explain duplication of distance at the example prefix sums.
22. Explain the barrier rule; explain the flag rules.
23. Describe the tree barrier.
24. Describe the symmetric dissemination barrier.



## Check your knowledge (3)

### Data parallelism

25. Explain how list ends are found in parallel.
26. Show iteration spaces for given loops and vice versa.
27. Explain which dependence vectors may occur in sequential (parallel) loops.
28. Explain the SRP transformations.
29. How are the transformation matrices used?
30. Which transformations can be used to parallelize the inner loop if the dependence vectors are  $(0,1)$  and  $(1,0)$ ?
31. How are bounds of nested loops described formally?

### Asynchronous messages

32. Explain the notion of a channel and its operations.
33. Explain typical channel structures.
34. Explain channel structures for the client/server paradigm.
35. What problem occurs if server processes receive each from several channels?
36. Explain the notion of conversation sequences.

## Check your knowledge (4)

37. Which operations does a node execute when it is part of a broadcast in a net?
38. Which operations does a node execute when it is part of a probe-and-echo?
39. How many messages are sent in a probe-and-echo scheme?

### Messages in distributed systems

40. Explain the worker paradigm.
41. Describe the process interface for distributed branch-and-bound.
42. Explain the technique for termination in a ring.

### Synchronous messages

43. Compare the fundamental notions of synchronous and asynchronous messages.
44. Explain the constructs for selective wait with synchronous messages.
45. Why are programs based on synchronous messages more compact and less redundant than those with asynchronous messages?
46. Describe a server for resource allocation according to the scheme for synchronous messages.

## Check your knowledge (5)

### Concurrent and functional programming

47. Explain why paradigms in functional and concurrent programming match well.
48. What are benefits of stream programming?
49. Compare implementations of the Sieve of Eratosthenes using streams or CSP.
50. Explain concurrency in Erlang, in particular selective receive.
51. Explain the characteristics of Scala, in particular its Actors.