C-1.1

Compilation Methods

Prof. Dr. Uwe Kastens

Summer 2013

C-1.2

1 Introduction

Objectives

The students are going to learn

- what the main tasks of the **synthesis part of optimizing compilers** are,
- how **data structures and algorithms** solve these tasks systematically,
- what can be achieved by **program analysis and optimizing transformations**,

Prerequisites

- Constructs and properties of programming languages
- What does a compiler know about a program?
- How is that information represented?
- Algorithms and data structures of the analysis parts of compilers (frontends)

Main aspects of the lecture ***Programming Languages and Compilers*** (PLaC, BSc program)
http://ag-kastens.upb.de/lehre/material/plac

Objectives:

The objectives of the course

In the lecture:

The objectives are explained.

Questions:

- What are your objectives?
- Do they match to these objectives?

Syllabus

Week	Chapter	Topic
1	1 Introduction	Compiler structure
	2 Optimization	Overview: Data structures, program transformations
2		Control-flow analysis
3		Loop optimization
4, 5		Data-flow analysis
6		Object oriented program analysis
7	3 Code generation	Storage mapping
		Run-time stack, calling sequence
8		Translation of control structures
9		Code selection by tree pattern matching
10, 11	4 Register allocation	Expression trees (Sethi/Ullman)
		Basic blocks (Belady)
		Control flow graphs (graph coloring)
12	5 Code Parallelization	Data dependence graph
13		Instruction Scheduling
14		Loop parallelization
15	Summary	

Lecture Compilation Methods SS 2013 / Slide 104

Objectives:

Overview over the topics of the course

In the lecture:

Comments on the topics

References

Course material:

Compilation Methods: http://ag-kastens.upb.de/lehre/material/compii
Programming Languages and Compilers: http://ag-kastens.upb.de/lehre/material/plac

Books:

U. Kastens: **Übersetzerbau**, Handbuch der Informatik 3.3, Oldenbourg, 1990; (sold out)

K. Cooper, L. Torczon: Engineering A Compiler, Morgan Kaufmann, 2003

S. S. Muchnick: **Advanced Compiler Design & Implementation**,
Morgan Kaufmann Publishers, 1997

A. W. Appel: **Modern Compiler Implementation in C**, 2nd Edition
Cambridge University Press, 1997, (in Java and in ML, too)

W. M. Waite, L. R. Carter: **An Introduction to Compiler Construction,**
Harper Collins, New York, 1993

M. Wolfe: **High Performance Compilers for Parallel Computing**, Addison-Wesley, 1996

A. V. Aho, M. S. Lam, R. Sethi, J. D. Ullman: **Compilers - Principles, Techniques, & Tools**,
2nd Ed, Pearson International Edition (Paperback), and Addison-Wesley, 2007

Lecture Compilation Methods SS 2013 / Slide 105

Objectives:

Useful books and electronic material in the web

In the lecture:

Comments on the items:

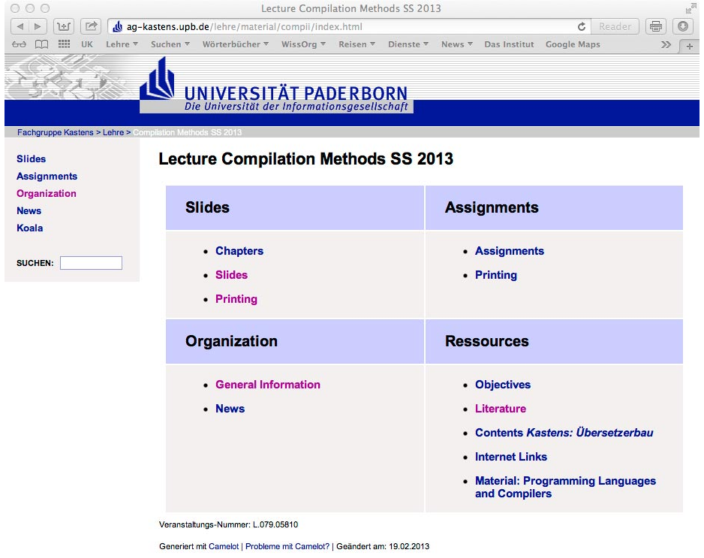
- The material for this course is available.
- The material of "Programming Languages and Compilers" (every winter semester) is a prerequisite for this course.
- The book "Übersetzerbau" isn't sold anymore. It is available in the library.
- The book by Muchnick contains very deep and concrete treatment of most important topics for optimizing compilers.

Questions:

- Find the referenced material in the web, become familiar with its structure, and set bookmarks for it.

C-1.6

Course Material in the Web: HomePage



Lecture Compilation Methods SS 2013 / Slide 106

Objectives:

The root page of the course material.

In the lecture:

The navigation structure is explained.

Assignments:

Explore the navigation structure.

C-1.6a

Course Material in the Web: Organization

Lecturer

Prof. Dr. Uwe Kastens:

Office hours

- Wed 16.00 - 17.00 F2.308
- Thu 11.00 - 12.00 F2.308

Hours

Lecture

- V2 Fr 11:15 - 12:45 F1.110

Start date: Fr Apr 12, 2013

Tutorials

- Ü2 Fr 13:15 - 14:45, F1.110, even weeks

Dates: 19.04., 03.05., 17.05., 31.05., 14.06., 28.06., 12.07.

Examination

This course is examined in an oral examination, which in general is held in English. It may be held in German, if the candidate does not need the certificate of an English examination.
In the study program Master of Computer Science the examination for this course is part of a module examination which covers two courses. It may contribute to the module examination of one of the modules III.1.2 (type A), III.1.5 (type A), or III.1.6 (type B). Please follow the instructions for examination registration
or in German zur Prüfungsanmeldung

In other study programs a single oral examination for this course may be taken.

In any case a candidate has to register for the examination in PAUL and has to ask for a date for the exam via eMail to me.

The next time spans I offer for oral exams are July 31 to Aug 01, 2013, and Oct 09 to 11, 2013.

Homework

Homework assignments

- Homework assignments are published every other week on Fridays.

Lecture Compilation Methods SS 2013 / Slide 106a

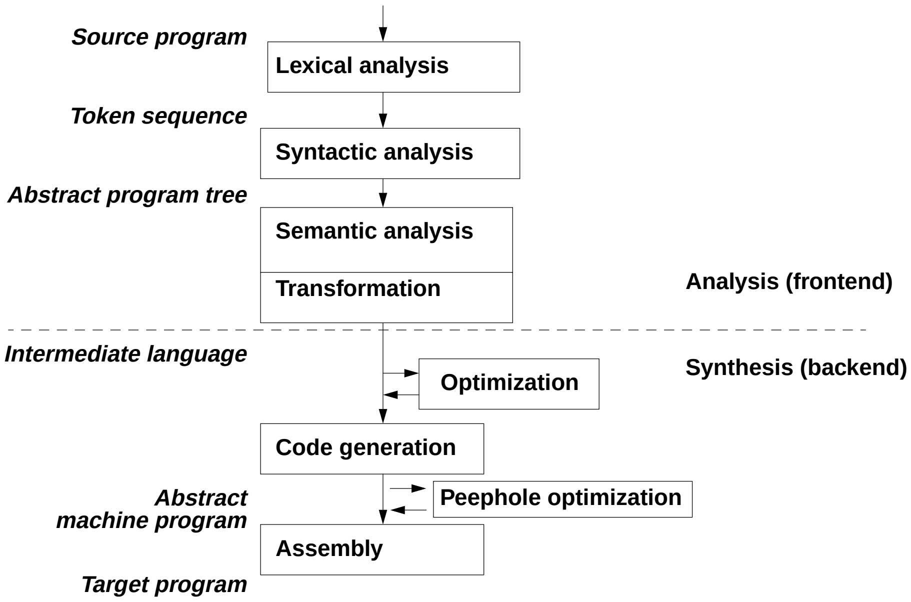
Objectives:

Agree on organizational items

In the lecture:

Check organizational items

Compiler Structure and Interfaces



Lecture Compilation Methods SS 2013 / Slide 107

Objectives:

Recall compiler structure and interfaces

In the lecture:

In this course we focus on the synthesis phase (backend).

Suggested reading:

Kastens / Übersetzerbau, Section 2.1

Assignments:

Compare this slide with U-08 and learn the translations of the technical terms used here.

2 Optimization

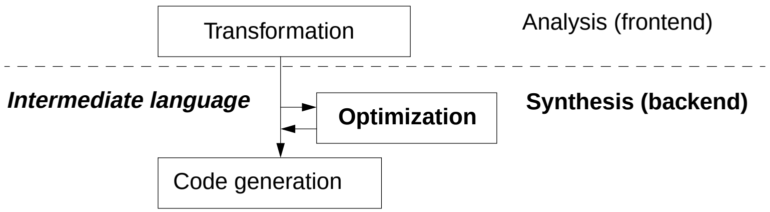
Objective:

Reduce run-time and / or code size of the program,
without changing its observable effects.
Eliminate redundant computations, simplify computations.

Input: Program in intermediate language

Task: find redundancies (**analysis**)
improve the code (**optimizing transformations**)

Output: Improved program in intermediate language



Lecture Compilation Methods SS 2013 / Slide 201

Objectives:

Overview over optimization

In the lecture:

- Program analysis computes safe assertions at compile time about execution of the program.
- Conventionally this phase is called "Optimization", although in most cases a formal optimum can not be defined or achieved with practical effort.

Suggested reading:

Kastens / Übersetzerbau, Section 8

Questions:

Give examples for observable effects that may not be changed.

Overview on Optimizing Transformations

Name of transformation: | **Example for its application:**

1. **Algebraic simplification** of expressions
   `2*3.14 => 6.28   x+0 => x   x*2 =>` shift left `x**2 => x*x`
2. **Constant propagation** (dt. Konstantenweitergabe)
   constant values of variables propagated to uses: `x = 2; ... y = x * 5;`
3. **Common subexpressions** (gemeinsame Teilausdrücke)
   avoid re-evaluation, if values are unchanged `x = a*(b+c);...y = (b+c)/2;`
4. **Dead variables** (überflüssige Zuweisungen)
   eliminate redundant assignments `x = a + b; ... x = 5;`
5. **Copy propagation** (überflüssige Kopieranweisungen)
   substitute use of x by y `x = y; ... ; z = x;`
6. **Dead code** (nicht erreichbarer Code)
   eliminate code, that is never executed `b = true;...if (b) x = 5; else y = 7;`

Lecture Compilation Methods SS 2013 / Slide 202

Objectives:

Get an idea of important transformations

In the lecture:

- The transformations are explained.
- The preconditions are discussed for some of them.

Suggested reading:

Kastens / Übersetzerbau, Section 8.1

Assignments:

- Apply as many transformations as possible in a given example program.

Questions:

- Which of the transformations need to analyze pathes through the program?
- Give an example for a pair of transformations, such that an application of the first one enables an application of the second.

Overview on Optimizing Transformations (continued)

Name of transformation: | **Example for its application:**

7. **Code motion** (Code-Verschiebung)
   move computations to cheaper places `if (c) x = (a+b)*2; else x = (a+b)/2;`
8. **Function inlining** (Einsetzen von Aufrufen)
   substitute call of small function by a computation over the arguments
   ```
   int Sqr (int i) { return i * i; }
   x = Sqr (b*3)
   ```
9. **Loop invariant code**
   move invariant code before the loop `while (b) {... x = 5; ...}`
10. **Induction variables in loops**
    transform multiplication into incrementation `i = 1; while (b) { k = i*3; f(k); i = i+1;}`

Lecture Compilation Methods SS 2013 / Slide 202a

Objectives:

Get an idea of important transformations

In the lecture:

- The transformations are explained.
- The preconditions are discussed for some of them.

Suggested reading:

Kastens / Übersetzerbau, Section 8.1

Assignments:

- Apply as many transformations as possible in a given example program.

Questions:

- Which of the transformations need to analyze pathes through the program?
- Give an example for a pair of transformations, such that an application of the first one enables an application of the second.

Program Analysis for Optimization

Static analysis:
static properties of program structure and of **every execution**;
safe, pessimistic assumptions
where input and dynamic execution paths are not known

Context of analysis - the larger the more information:

Expression	local optimization	
Basic block	local optimization	
procedure (control flow graph)	global	intra-procedural optimization
program module (call graph) separate compilation	global	inter-procedural optimization
complete program	optimization at link-time or at run-time	

Analysis and Transformation:
Analysis provides preconditions for **applicability of transformations**

Transformation may change analysed properties,
may **inhibit or enable** other transformations

Order of analyses and transformations **is relevant**

Lecture Compilation Methods SS 2013 / Slide 203

Objectives:
Overview over optimization

In the lecture:
- Program analysis computes safe assertions at compile time about execution of the program.
- The larger the analysis context, the better the information, the more positions where transformations are applicable.

Suggested reading:
Kastens / Übersetzerbau, Section 8

Program Analysis in General

Program text is systematically analyzed to exhibit
structures of the program,
properties of program entities,
relations between program entities.

Objectives:

Compiler:
- Code improvement
- automatic parallelization
- automatic allocation of threads

Software engineering tools:
- program understanding
- software maintenance
- evaluation of software qualities
- reengineering, refactoring

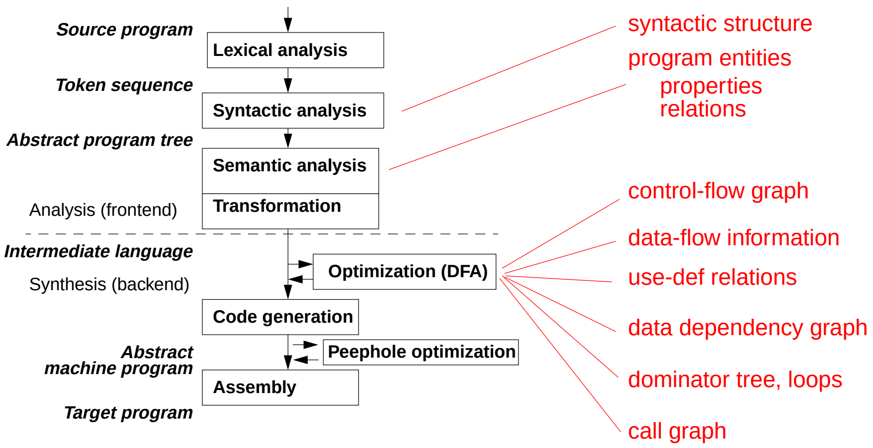
Methods for program analysis stem from **compiler construction**

Lecture Compilation Methods SS 2013 / Slide 204

Objectives:
Program analysis beyond optimization

In the lecture:
Examples are given for the objectives

Overview on Program Analysis in Compilers



Lecture Compilation Methods SS 2013 / Slide 205

Objectives:

Analysis methods in compiler structure

In the lecture:

The topics on the slide are explained.

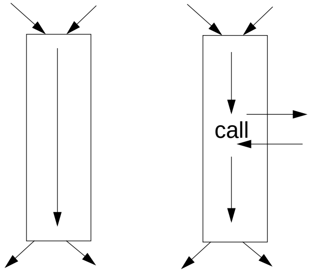
Basic Blocks

Basic Block (dt. Grundblock):
Maximal sequence of instructions that can be entered only at the first of them and exited only from the last of them.

Begin of a basic block:

- procedure entry
- target of a branch
- instruction after a branch or return (must have a label)

Function calls are usually not considered as a branch, but as operations that have effects



Local optimization
considers the context of one single basic block (or part of it) at a time.

Global optimization:
Basic blocks are the nodes of control-flow graphs.

Lecture Compilation Methods SS 2013 / Slide 206

Objectives:

Understand the notion of basic blocks

In the lecture:

The topics on the slide are explained. Examples are given.

- The definition is explained.
- The construction is explained using the example of C-2.7.
- The consequences of having calls in a basic block are discussed.

Questions:

- Explain the decomposition of intermediate code into basic blocks for C-2.7 and for further examples.

Example for Basic Blocks

A C function that computes Fibonacci numbers:

```
int fib (int m)
{  int f0 = 0, f1 = 1, f2, i;
   if (m <= 1)
      return m;
   else
   {  for(i=2; i<=m; i++)
      {  f2 = f0 + f1;
         f0 = f1;
         f1 = f2;
      }
      return f2;
} }
```

Intermediate code with basic blocks:

[Muchnick, p. 170]

```
    receive m                B1
    f0 <- 0
    f1 <- 1
    if m <= 1 goto L3

    i <- 2                   B3

L1: if i <= m goto L2        B4

    return f2                B5

L2: f2 <- f0 + f1            B6
    f0 <- f1
    f1 <- f2
    i <- i + 1
    goto L1

L3: return m                 B2
```

if-condition belongs to the preceding basic block

while-condition does not belong to the preceding basic block

Lecture Compilation Methods SS 2013 / Slide 207

Objectives:

Example for the construction of basic blocks

In the lecture:

The decomposition into basic blocks is explained according to C-2.6 using the example.

Control-Flow Graph (CFG)

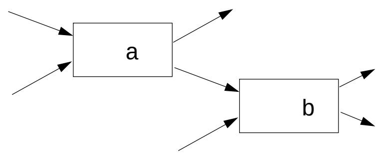
A **control-flow graph, CFG** (dt. Ablaufgraph)
represents the control structure of a function

Nodes: **basic blocks** and 2 unique nodes **`entry`** and **`exit`**.

Edge a -> b: **control may flow** from the end of **a** to the begin of **b**

Fundamental data structure for

- control flow analysis
- structural transformations
- code motion
- data-flow analysis (DFA)



Lecture Compilation Methods SS 2013 / Slide 208

Objectives:

Understand the notion of control-flow graphs

In the lecture:

Examples are given.

- The definition is explained.
- The example of C-2.9 is explained.
- The representation of loops in control-flow graphs is compared to source language representation.
- Algorithms that recognize loops in control-flow graphs are presented in the next section.

Questions:

- Why is the loop structure of source programs not preserved on the level of intermediate languages?

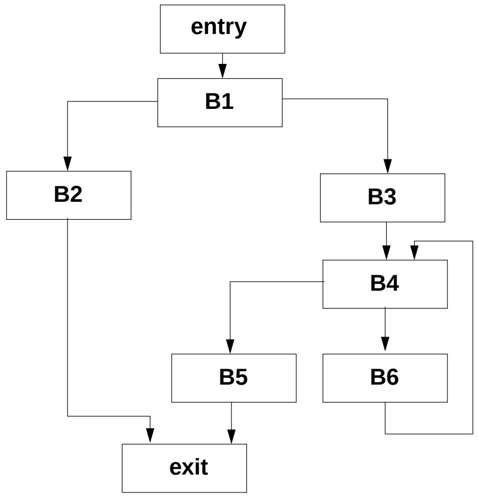
Example for a Control-flow Graph

Intermediate code with basic blocks:

```
    receive m                 B1
    f0 <- 0
    f1 <- 1
    if m <= 1 goto L3

    i <- 2                    B3

L1: if i <= m goto L2         B4

    return f2                 B5

L2: f2 <- f0 + f1             B6
    f0 <- f1
    f1 <- f2
    i <- i + 1
    goto L1

L3: return m                  B2
```

Control-flow graph:

[Muchnick, p. 172]



Lecture Compilation Methods SS 2013 / Slide 209

Objectives:

Example for a control-flow graph

In the lecture:

The control-flow graph represents the basic blocks and their branches, as defined in C-2.8.

Questions:

Control-Flow Analysis

Compute **properties on the control-flow** based on the CFG:

- **dominator relations**:
  properties of paths through the CFG
- **loop recognition**:
  recognize loops - independent of the source language construct
- **hierarchical reduction of the CFG**:
  a region with a unique entry node on the one level is a node of the next level graph

Apply **transformations** based on control-flow information:

- **dead code elimination**:
  eliminate unreachable subgraphs of the CFG
- **code motion**:
  move instructions to better suitable places
- **loop optimization**:
  loop invariant code, strength reduction, induction variables

Lecture Compilation Methods SS 2013 / Slide 210

Objectives:

Overview on control-flow analysis

In the lecture:

The basic ideas of the analysis and transformation techniques are given.

Suggested reading:

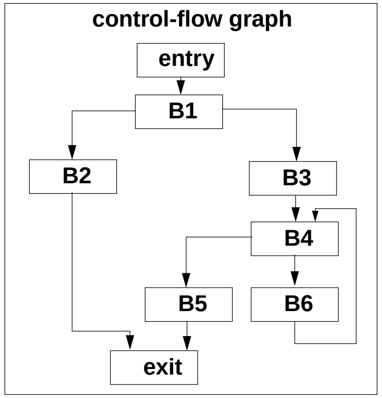
Kastens / Übersetzerbau, Section 8.2.1

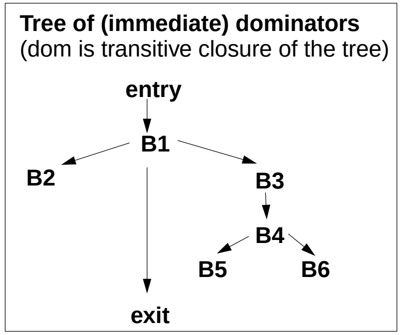
Dominator Relation on CFG

Relation over nodes of a CFG, characterizes paths through CFG,
used for loop recognition, code motion

a dominates b (**a dom b**):
a is on every path from the entry node to b (reflexive, transitive, antisymmetric)

a is immediate dominator of b (a idom b):
a dom b and a ≠ b, and there is no c such that c ≠ a, c ≠ b, a dom c, c dom b.





Lecture Compilation Methods SS 2013 / Slide 211

Objectives:

Understand the dominator relation

In the lecture:

Explain

- the definitions,
- the example.

Suggested reading:

Kastens / Übersetzerbau, Section 8.2.2

Questions:

- How is the dominator relation obtained from the immediate dominator relation.
- Why is the dominator relation useful for code motion?

Immediate Dominator Relation is a Tree

Every node has a unique immediate dominator.

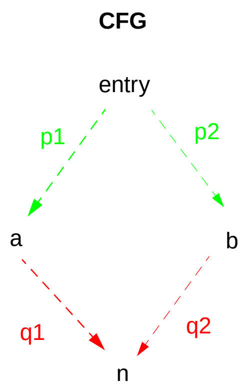
The dominators of a node are linearly ordered by the idom relation.

Proof by contradiction:
Assume:
a ≠ b, a dom n, b dom n and
not (a dom b) and not (b dom a)

Then there are pathes in the CFG

- p1: from entry to a not touching b, since not (b dom a)
- p2: from entry to b not touching a, since not (a dom b)
- q1: from a to n not touching b, since a dom n and not (a dom b)
- q2: from b to n not touching a, since b dom n and not (b dom a)

Hence, there is a path p1-q1 from
entry via a to n not touching b.
That is a contradiction to the assumption b dom n.
Hence, n has a unique immediate dominator, either a or b.



Lecture Compilation Methods SS 2013 / Slide 211a

Objectives:

The set of dominators of a node is ordered

In the lecture:

The proof is explained.

Dominator Computation

Algorithm computes the sets of dominators Domin(n) for all nodes n∈N of a CFG:

```
for each n∈N do Domin(n) = N;
Domin(entry) = {entry};

repeat
   for each n∈N-{entry} do
      T = N;
      for each p∈pred(n) do
         T = T ∩ Domin(p);
      Domin(n) = {n} ∪ T;
until Domin is unchanged
```

Symmetric relation for backward analysis:

a postdominates b (a pdom b):
a is on every path from b to the exit node (reflexive, transitive, antisymmetric)

Lecture Compilation Methods SS 2013 / Slide 212

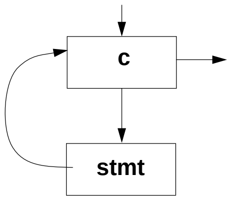
Objectives:
Understand the algorithm

In the lecture:
The algorithm is explained using the example of C-2.11

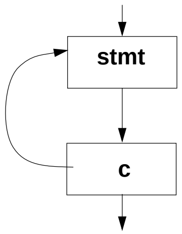
Questions:
What properties and transformations can be characterized using the postdominator relation?

Loop Recognition: Structured Loops

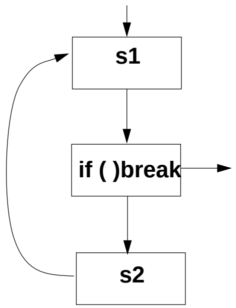
while (c) stmt;



do stmt; while (c);



do s1; if ( )break; s2; while (true);



Lecture Compilation Methods SS 2013 / Slide 213

Objectives:
Comm on loop structures

In the lecture:
Explain

- the loop structures,
- their occurrences in programming languages,

to get an intuitive understandig of loops;

Suggested reading:
Kastens / Übersetzerbau, Section 8.2.2

Loop Recognition: Natural Loops

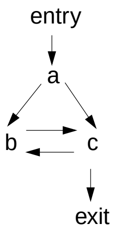
Back edge t->h in a CFG: head h dominates tail t (h dom t).

Natural loop of a back edge t->h:
set S of nodes such that S contains h, t and
all nodes from which t can be reached without passing through h.
h is the **loop header**.

Iterative computation of the natural loop for t->h:
add predecessors of nodes in S according to the formula:

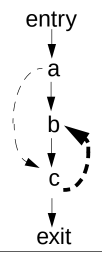
$$S = \{h, t\} \cup \{ p \mid \exists a\ (a \in S \setminus \{h\} \wedge p \in pred(a)) \}$$

This definition of **back edges** is stronger than that of **DFS back edges**:



no back edge

a DFS classification of edges:
tree, forward, back



DFS back edge

Lecture Compilation Methods SS 2013 / Slide 213a

Objectives:
Notion of natural loops

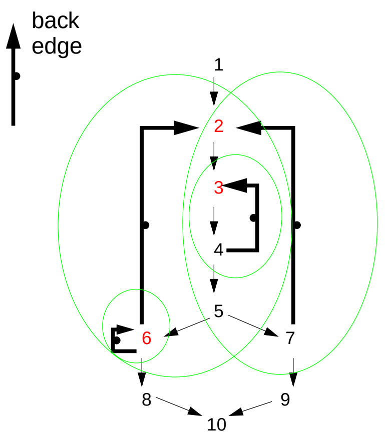
In the lecture:
- Explain the definitions;
- give an intuitive understandig of loops;
- show patterns for while and repeat loops, and for loop exit;
- discuss the example of C-2.14.

Suggested reading:
Kastens / Übersetzerbau, Section 8.2.2

Questions:
- What is the role of the loop header?
- Why can't the graph on the left been derived from structured loops?

Example for Loop Recognition

back edge:	natural loop:
4 -> 3	$S_1 = \{3,4\}$
6 -> 2	$S_2 = \{2, 3, 4, 5, 6\}$
7 -> 2	$S_3 = \{2, 3, 4, 5, 7\}$
6 -> 6	$S_4 = \{6\}$



loops are
- **disjoint** $S_1 \cap S_4 = \emptyset$
- **nested** $S_1 \subset S_2$
- **non-nested,** S_2, S_3
but have the same loop header,
are comprised into one loop

Lecture Compilation Methods SS 2013 / Slide 214

Objectives:
Recognize natural loops

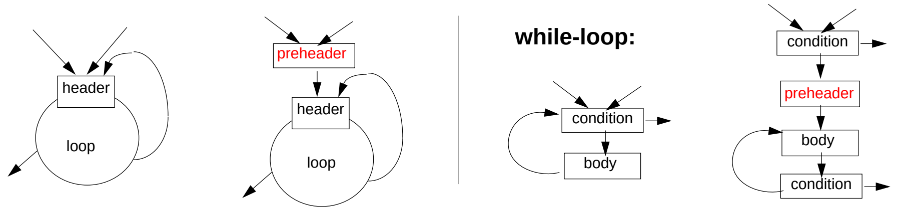
In the lecture:
- Apply the definitions of C-2.13a to this example;
- discuss nesting of loops.

Suggested reading:
Kastens / Übersetzerbau, Section 8.2.2

Questions:
- Can you give a program structure with repeat-loops, loop-exits, and if-statements for this graph, such that loop S2 is nested in S3?

Loop Optimization

- Introduce a **preheader** for a loop, as a place for loop invariant computations:
  a new, empty basic block that lies on every path to the loop header, but is not iterated:



- move **loop invariant computations** to the preheader:
  check use-def-chains: if an expression E contains no variables that are defined in the loop, then replace E by a temporary variable t, and compute t = E; in the preheader.
- eliminate **redundant bounds-checks**:
  propagate value intervals using the same technique as for constant propagation (see DFA)
  Example in Pascal:

```
var   a: array [1..10] of integer;
      i: integer;

for i := 1 to 10 do a[i] := i;
```

- **induction variables**, **strength reduction**: see next slide

Lecture Compilation Methods SS 2013 / Slide 215

Objectives:

Get an idea of loop otimization

In the lecture:

- while-loops have to be transformed into repeat-loops, before adding a preheader.
- A use-def-chain links an ocurrence of a variable where it is read (used) to all occurrences where it is written (defined) such that the value may propagate to this point of use. us-def-chains are a result of data flow analysis.
- Explain the optimization techniques.

Suggested reading:

Kastens / Übersetzerbau, Section 8.2.3

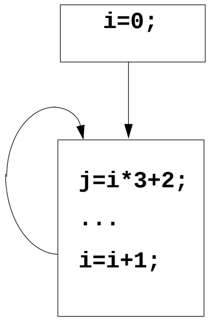
Loop Induction Variables

Induction variables may occur in any loop - not only in `for` loops.

Induction variable i:
i is incremented (decremented) by a constant value c on every iteration.

Basic induction variable i:
There is exactly one definition `i = i + c;` or `i = i - c;`
that is executed on every path through the loop.

Dependent induction variable j:
j depends on induction variable i by a
linear function `i * a + b`
represented by (i, a, b).



Lecture Compilation Methods SS 2013 / Slide 216

Objectives:

Understand the notion of induction variables

In the lecture:

Explain how

- induction variables depend on each other

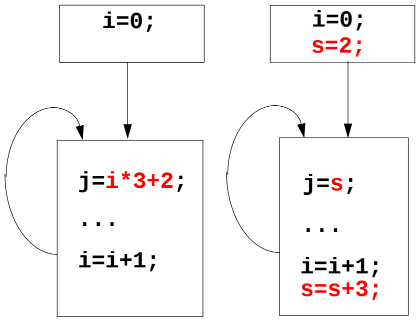
Suggested reading:

Kastens / Übersetzerbau, Section 8.3.4

Transformation of Induction Variables

Transformation of dependent induction variables:

1. For each (i, a, b) create a temporary variable s. `j: (i, 3, 2)`
2. Initialize `s = i * a + b;` in the preheader.
3. Replace `i * a + b` in the loop by s.
4. Add `s = s + c*a;` behind the increment of i



Strength reduction:
Replace a costly operation (multiplication) by a cheaper one (addition).

Linear increment of array address computation (next slide)

Lecture Compilation Methods SS 2013 / Slide 217

Objectives:

Understand the notion of induction variables

In the lecture:

Explain how

- induction variables are transformed.

Suggested reading:

Kastens / Übersetzerbau, Section 8.3.4

Questions:

- How is the technique applied to array indexing?

Examples for Transformations of Induction Variable

```
do
   k = i*3+1;
   f (5*k);
   /* x = a[i]; compiled: */
   x = cont(start+i*elsize);
   i = i + 2;
while (Ek)
```

basic induction variable:

```
i:   c = 2
```

dependent induction variables:

```
k:   (i, 3, 1)
arg: (k, 5, 0)
ind: (i, elsize, start)
```

```
sk = i*3+1;
sarg = sk*5;
sind = start + i*elsize;
do
   k = sk;
   f (sarg);
   x = cont (sind);
   i = i + 2;
   sk = sk + 6;
   sarg = sarg + 30;
   sind = sind + 2*elsize;
while (Ek)
```


Lecture Compilation Methods SS 2013 / Slide 217a

Objectives:

Apply the transformation pattern

In the lecture:

The examples are explained:

- expressions linear in induction variables can be transformed, e. g. function arguments;
- multiplications in array addresses are replaced by incrementation.

Data-Flow Analysis

Data-flow analysis (DFA) provides information about
how the **execution of a program may manipulate its data**.

Many different problems can be formulated as **data-flow problems**, for example:

- Which assignments to variable **v** may influence a use of **v** at a certain program position?
- Is a variable **v** used on any path from a program position **p** to the exit node?
- The values of which expressions are available at program position **p**?

Data-flow problems are stated in terms of

- **paths through the control-flow graph** and
- **properties of basic blocks**.

Data-flow analysis provides information for **global optimization.**

Data-flow analysis does not know

- which input values are provided at run-time,
- which branches are taken at run-time.

Its results are to be interpreted **pessimistic**

Lecture Compilation Methods SS 2013 / Slide 218

Objectives:

Goals and ability of data-flow analysis

In the lecture:

- Examples for the use of DFA information are given.
- Examples for pessimistic information are given.

Suggested reading:

Kastens / Übersetzerbau, Section 8.2.4

Questions:

- What's wrong about optimistic information?
- Why can pessimistic information be useful?

Data-Flow Equations

A data-flow problem is stated as a **system of equations** for a control-flow graph.

System of Equations for **forward problems** (propagate information along control-flow edges):

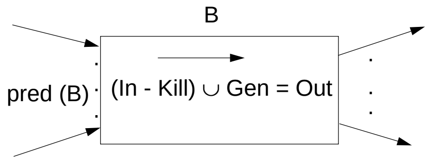
Example **Reaching definitions:**
A definiton **d** of a variable **v** reaches the begin of a block **B** if **there is a path** from **d** to **B** on which **v** is not assigned again.

In, Out, Gen, Kill represent analysis information:
sets of statements,
sets of variables,
sets of expressions
depending on the analysis problem

2 equations for each basic block:

$$Out(B) = f_B(In(B))$$
$$= Gen(B) \cup (In(B) - Kill(B))$$

$$In(B) = \underset{h \in pred(B)}{\Theta} Out(h)$$



In, Out **variables** of the system of equations for each block

Gen, Kill a pair of **constant sets** that characterize a block w.r.t. the DFA problem

Θ meet operator; e. g. Θ **=** $\cup$ for „reaching definitions“, Θ **=** $\cap$ for „available expressions“

Lecture Compilation Methods SS 2013 / Slide 219

Objectives:

A DFA problem is modeled by a system of equations

In the lecture:

- The equation pattern is explained.
- Equations are defined over sets.
- In this example: sets of assignment statements at certain program positions.
- The meet operator being the union operator is correlated to "there is a path" in the problem statement.
- Note: In this context a "definition of a variable" means an "assignment of a variable".

Suggested reading:

Kastens / Übersetzerbau, Section 8.2.4

Questions:

- Explain the meaning of In(B)= {d1: x=5, d4: x=7, d6: y=a+1} for a particular block B.

Specification of a DFA Problem

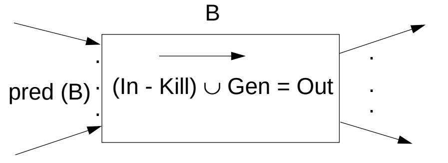
Specification of reaching definitions:

1. **Description**:
   A definiton **d** of a variable **v** reaches the begin of a block **B**
   if **there is a path** from **d** to **B** on which **v** is not assigned again.
2. It is a **forward problem**.
3. The **meet operator** is union.
4. The **analysis information** in the sets are assignments at certain program positions.
5. **Gen (B)**:
   contains all definitions **`d: v = e;`** in **B**,
   such that **v** is not defined after **d** in **B**.
6. **Kill (B)**:
   if **v** is assigned in **B**, then **Kill(B)**
   contains all definitions **`d: v = e;`**
   of blocks different from **B**.

2 equations for each basic block:

$$\text{Out}(B) = f_B(\text{In}(B)) = \text{Gen}(B) \cup (\text{In}(B) - \text{Kill}(B))$$

$$\text{In}(B) = \underset{h \in \text{pred}(B)}{\Theta} \text{Out}(h)$$



Lecture Compilation Methods SS 2013 / Slide 220

Objectives:

Specify a DFA problem systematically

In the lecture:

- The items that characterize a DFA problem are explained.
- The definition of Gen and Kill is explained.

Suggested reading:

Kastens / Übersetzerbau, Section 8.2.4

Questions:

- Why does this definition of Gen and Kill serves the purpose of the description in the first item?

Variants of DFA Problems

- **forward** problem:
  DFA information flows **along the control flow**
  In(B) is determined by Out(h) of the predecessor blocks

  backward problem (see C-2.23):
  DFA information flows **against the control flow**
  Out(B) is determined by In(h) of the successor blocks
- **union** problem:
  problem description: „there is a path“;
  meet operator is $\Theta = \cup$
  solution: minimal sets that solve the equations

  intersect problem:
  problem description: „for all paths“
  meet operator is $\Theta = \cap$
  solution: maximal sets that solve the equations
- **optimization information**: **sets of** certain statements, of variables, of expressions.

Further classes of DFA problems over general lattices instead of sets are not considered here.

Lecture Compilation Methods SS 2013 / Slide 221

Objectives:

Summary of the DFA variants

In the lecture:

- The variants of DFA problems are compared.

Suggested reading:

Kastens / Übersetzerbau, Section 8.2.4

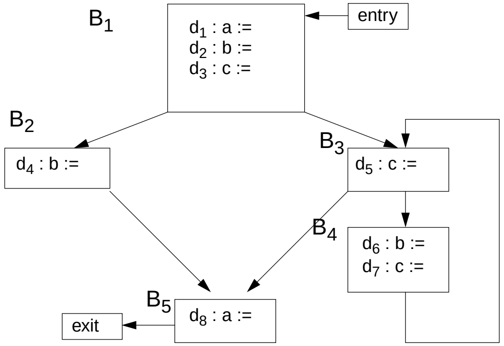
Questions:

- Explain the relation of the meet operator, the paths in the graph, and the maximal / minimal solutions.

Example Reaching Definitions

Gen (B):
contains all definitions **`d: v = e;`**
in **B**, such that **v** is not defined
after **d** in **B**.

Kill (B):
contains all definitions **`d: v = e;`**
in blocks different from **B**,
such that B has a definition of **v**.



	Description of DFA-Problem: Gen	Kill	DFA-Solution: In	Out
B_1	d_1, d_2, d_3	d_4, d_5, d_6, d_7, d_8	$\emptyset$	d_1, d_2, d_3
B_2	d_4	d_2, d_6	d_1, d_2, d_3	d_1, d_3, d_4
B_3	d_5	d_3, d_7	d_1, d_2, d_3, d_6, d_7	d_1, d_2, d_5, d_6
B_4	d_6, d_7	d_2, d_3, d_4, d_5	d_1, d_2, d_5, d_6	d_1, d_6, d_7
B_5	d_8	d_1	$d_1, d_2, d_3, d_4, d_5, d_6$	$d_2, d_3, d_4, d_5, d_6, d_8$

Lecture Compilation Methods SS 2013 / Slide 222

Objectives:

Understand the meaning of DFA sets

In the lecture:

- The example for C-2.20 is explained.

Suggested reading:

Kastens / Übersetzerbau, Section 8.2.4

Questions:

- Check that the In and Out sets solve the equations for the CFG.
- How can you argue that the solution is minimal?
- Add some elements to the solution such that it still solves the equations. Explain what such non-minimal solutions mean.

Iterative Solution of Data-Flow Equations

Input: the CFG; the sets Gen(B) and Kill(B) for each basic block B
Output: the sets In(B) and Out(B)

Algorithm:
```
repeat
   stable := true;
   for all B ≠ entry   {*}
   do begin
      for all V ∈ pred(B) do
         In(B):= In(B) Θ Out(V);
      oldout:= Out(B);
      Out(B):= Gen(B) ∪ (In(B)-Kill(B));
      stable:= stable and Out(B)=oldout
   end
until stable
```

Initialization

Union: empty sets
```
for all B do
begin
   In(B):=∅;
   Out(B):=Gen(B)
end;
```

Intersect: full sets
```
for all B do
begin
   In(B) := U;
   Out(B):=
      Gen(B)∪
         (U - Kill(B))
end;
```

Complexity: $O(n^3)$ with n number of basic blocks
$O(n^2)$ if $|$**pred (B)**$|$ $\leq$ **k << n** for all B

Lecture Compilation Methods SS 2013 / Slide 222b

Objectives:

Understand the iterative DFA algorithm

In the lecture:

The topics on the slide are explained. Examples are given.

- Initialization variants are explained.
- The algorithm is explained.

Suggested reading:

Kastens / Übersetzerbau, Section 8.2.5, 8.2.6

Questions:

- How is the initialization related to the size of the solution for the two variants union and intersect?
- Why does the algorithm terminate?

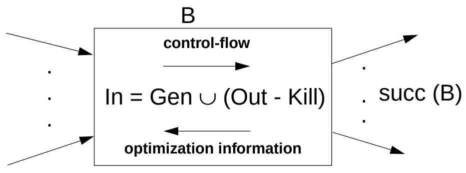
Backward Problems

System of Equations for **backward problems**
propagate information against control-flow edges:

2 equations for each basic block:

$$In(B) = f_B(Out(B)) = Gen(B) \cup (Out(B) - Kill(B))$$

$$Out(B) = \underset{h \in succ(B)}{\Theta} In(h)$$



Example **Live variables:**

1. Description: Is variable **v** alive at a given point **p** in the program, i. e. **is there a path** from **p** to the exit where **v** is used but not defined before the use?
2. backward problem
3. optimization information: sets of variables
4. meet operator: Θ **=** $\cup$ union
5. Gen (B): variables that are used in B, but not defined before they are used there.
6. Kill (B): variables that are defined in B, but not used before they are defined there.

Lecture Compilation Methods SS 2013 / Slide 223

Objectives:

Symmetry of forward and backward schemes

In the lecture:

The topics on the slide are explained. Examples are given.

- The equation pattern is explained.
- The DFA problem "live variables" is explained.

Suggested reading:

Kastens / Übersetzerbau, Section 8.2.4

Questions:

- How do you determine the live variables **within** a basic block?

Important Data-Flow Problems

1. **Reaching definitions:** A definiton **d** of a variable **v** reaches the beginning of a block **B** if there is a path from **d** to **B** on which **v** is not assigned again.
   DFA variant: forward; union; set of assignments
   Transformations: use-def-chains, constant propagation, loop invariant computations
2. **Live variables:** Is variable **v** alive at a given point **p** in the program, i. e. there is a path from **p** to the exit where **v** is used but not defined before the use.
   DFA variant: backward; union; set of variables
   Transformations: eliminate redundant assignments
3. **Available expressions:** Is expression **e** computed on every path from the entry to a program position **p** and none of its variables is defined after the last computation before **p**.
   DFA variant: forward; intersect; set of expressions
   Transformations: eliminate redundant computations
4. **Copy propagation:** Is a copy assignment **c: x = y** redundant, i.e. on every path from **c** to a use of **x** there is no assignment to **y**?
   DFA variant: forward; intersect; set of copy assignments
   Transformations: remove copy assignments and rename use
5. **Constant propagation:** Has variable **x** at position **p** a known value, i.e. on every path from the entry to **p** the last definition of **x** is an assignment of the same known value.
   DFA variant: forward; combine function; vector of values
   Transformations: substitution of variable uses by constants

Lecture Compilation Methods SS 2013 / Slide 224

Objectives:

Recognize the DFA problem scheme

In the lecture:

- The DFA problems and their purpose are explained.
- The DFA classification is derived from the description.
- Examples are given.
- Problems like copy propagation oftem match to code that results from other optimizing transformations.

Suggested reading:

Kastens / Übersetzerbau, Section 8.3

Questions:

- Explain the classification of the DFA problems.
- Construct an example for each of the DFA problems.

Algebraic Foundation of DFA

DFA performs computations on a **lattice (dt. Verband)** of values, e. g. bit-vectors representing finite sets. It guarantees termination of computation and well-defined solutions. see [Muchnick, pp 223-228]

A **lattice L** is a set of values with two operations: $\cap$ **meet** and $\cup$ **join**

Required properties:

1. **closure**: $x, y \in L$ implies $x \cap y \in L$, $x \cup y \in L$
2. **commutativity**: $x \cap y = y \cap x$ and $x \cup y = y \cup x$
3. **associativity**: $(x \cap y) \cap z = x \cap (y \cap z)$ and $(x \cup y) \cup z = x \cup (y \cup z)$
4. **absorption**: $x \cap (x \cup y) = x = x \cup (x \cap y)$
5. unique elements **bottom** $\perp$, **top** T:
   $x \cap \perp = \perp$ and $x \cup T = T$

In most DFA problems only a **semilattice** is used with L, $\cap$, $\perp$ or L, $\cup$, T

Partial order defined by meet, $x \subseteq y$: $x \cap y = x$; defined by join: $x \supseteq y$: $x \cup y = x$
(transitive, antisymmetric, reflexive)

Lecture Compilation Methods SS 2013 / Slide 224a

Objectives:

Recall algebraic structure lattice

In the lecture:

The topics on the slide are explained using examples of C-2.24b

Some DFA Lattices

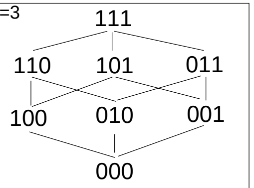
2 Bool

$\cap$ = and
$\cup$ = or

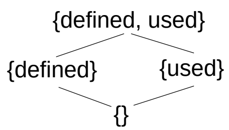
T = true
|
$\perp$ = false

1 Bit-Vector $BV^{n=3}$

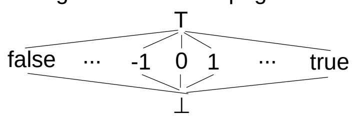
$\cap$ = bitwise and
$\cup$ = bitwise or



3 Variable usage



4 ICP Integer Constant Propagation Lattice



$n \cap \perp = \perp$ $n \cap n = n$ $n \cap m = \perp$ if $n \neq m$
$n \cup T = T$ $n \cup n = n$ $n \cup m = T$ if $n \neq m$

5 Range Lattice: $[lo, hi] \in (Z \cup \{-\infty, \infty\})^2$

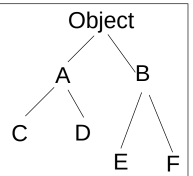
$\perp$ = [ ] empty range, T = $[-\infty, \infty]$,
$x \subseteq y$: x is contained in y

$\cap$: $[l1, h1] \cap [l2, h2] = x$
let l = max (l1, l2),
h = min (h1, h2),
x = if h < l then $\perp$ else [l, h]

$\cup$: $[l1, h1] \cup [l2, h2]$ =
[min(l1, l2), max(h1, h2)]

6 Semilattice of types

$\cup$: $x \cup y$ = smallest common supertype of x and y



Lecture Compilation Methods SS 2013 / Slide 224b

Objectives:

Most important DFA lattices

In the lecture:

- The Examples are explained.
- A new lattice can be constructed by elementwise composition of simpler lattices; e.g. a bit-vector lattice is an n-fold composition of the lattice Bool.
- A new lattice may be constructed for a particular DFA problem.

Monotone Functions Over Lattices

The **effects of program constructs on DFA information** are described by functions over a suitable lattice,

e. g. the function for basic block B_3 on C-2.22:

$f_3(<x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8>) = <x_1\ x_2\ 0\ x_4\ 1\ x_6\ 0\ x_8> \in BV^8$

Gen-Kill pair encoded as function

f: $L \rightarrow L$ is a **monotone function** over the lattice L if
$\forall\ x, y \in L: x \subseteq y \Rightarrow f(x) \subseteq f(y)$

Finite height of the lattice and **monotonicity** of the functions guarantee **termination** of the algorithms.

Fixed points z of the function f, with f(z) = z, is a solution of the set of DFA equations.

MOP: Meet over all paths solution is desired, i. e. the „best“ with respect to L

MFP: Maximum fixed point is computed by algorithms, if functions are monotone

If the functions f are additionally **distributive**, then **MFP = MOP**.
f: $L \rightarrow L$ is a **distributive function** over the lattice L if
$\forall\ x, y \in L: f(x \cap y) = f(x) \cap f(y)$

Lecture Compilation Methods SS 2013 / Slide 224c

Objectives:
DFA equations and monotone functions

In the lecture:
Understand solution of DFA equations as fixed point of monotone functions.

Variants of DFA Algorithms

Heuristic improvement:

Goal: propagate changes in the In and Out sets as fast as possible.
Technique: visit CFG nodes in topological order in the outer for-loop {*}.
Then the number of iterations of the outer repeat-loop is only determined by back edges in the CFG

Algorithm for backward problems:

Exchange In and Out sets symmetrically in the algorithm of C-2.22b.
The nodes should be visited in topological order as if the directions of edges were flipped.

Hierarchical algorithms, interval analysis:

Regions of the CFG are considered nodes of a CFG on a higher level.
That abstraction is recursively applied until a single root node is reached.
The Gen, Kill sets are combined in upward direction;
the In, Out sets are refined downward.

Lecture Compilation Methods SS 2013 / Slide 226

Objectives:
Overview on DFA algorithms

In the lecture:
- The variants of the algorithm of C-2.25 are explained.
- The improvement is discussed.
- The idea of hierarchical approaches is explained.

Suggested reading:
Kastens / Übersetzerbau, Section 8.2.5, 8.2.6

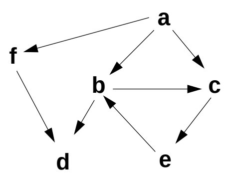
Questions:
- For a backward problem the blocks could be considered in reversed topological order. Why is that not a good idea?

Program Analysis: Call Graph (context-insensitive)

Nodes: defined functions

Arc g -> h: function g contains a call h(),
i. e. a call g() **may** cause the execution of a call h()

```
void a () {...b()...c()...f()...}
void b () {...d()...c()...}
void c()  {...e()...}
void d()  {...}
void e()  {...v++;...b()...}
void f()  {...d()...}
```



Analysis of structure:
b, c, e are recursive;
a, d, f are non-recursive

Propagation of properties:
assume a call e() may **modify a global variable** v
then calls a(), b(), c() may indirectly cause modification of v

```
v = f(); cnt = 0; while(...){...b(); cnt += v;}
```


Lecture Compilation Methods SS 2013 / Slide 227

Objectives:

Understand call graphs

In the lecture:

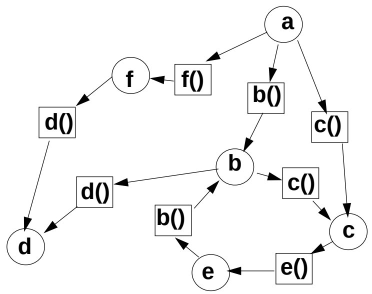
- Structural abstraction of call relation,
- Structural properties, e. g. reachability,
- Simplified implementation of non-recursive functions, of functions without calls, of functions that are never called.
- Propagation of information along call paths.
- Description of function behaviour, e. g. no side-effect on global variables.

Program Analysis: Call Graph (context-sensitive)

Nodes: defined functions and calls (bipartite)

Arc g -> h: function g contains a call h(),i.e a call g() **may** cause the execution of a call h()
or call g() leads to function g

```
void a () {...b()...c()...f()...}
void b () {...d()...c()...}
void c()  {...e()...}
void d()  {...}
void e()  {...v++;...b()...}
void f()  {...d()...}
```



Calls of the same function in different contexts are distinguished by **different nodes**, e.g. the call of c in a and in b.

Analysis can be **more precise** in that aspect.

Lecture Compilation Methods SS 2013 / Slide 227a

Objectives:

Understand context-sensitive call graphs

In the lecture:

Distinguish context-insensitive and context-sensitive call graphs

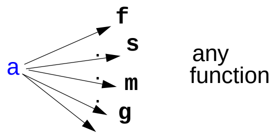
Calls Using Function Variables

Contents of **function variables is assigned at run-time**.

Static analysis does not know (precisely) which function is called.

Call graph has to assume that **any function may be called**.

```
void a()
  {...(*h)(0.3, 27)...}
```



Analysis for a better approximation of potential callees:

only those functions which

1. fit to the type of h
2. are assigned somewhere in the program
3. can be derived from the reaching definitions at the call

```
void m (int j) {...}

void g (float x, int i) {...}

...k = m;... f(g); ...

void a()
  {  void (*h)(float,int) = g;
     ...
     if(...) h = s;

     ...(*h)(0.3, 27)...
  }
```


Lecture Compilation Methods SS 2013 / Slide 228

Objectives:

Approximate call targets

In the lecture:

- Explain the approximation techniques using the example.
- Relate the problem to dynamically bound method calls.

Analysis of Object-Oriented Programs

Aspects specific for object-oriented analysis:

1. **hierarchy of classes and interfaces** specifies a complex **system of subtypes**
2. **hierarchy of classes and interfaces** specifies **inheritance and overriding** relation for methods
3. **dynamic method binding** for method calls `v.m(...)` the **callee is determined at run-time** good object-oriented style relies on that feature
4. **many small methods** are typical object-oriented style
5. **library use and reuse of modules** complete program contains many **unused classes and methods**

Static predictions for dynamically bound method calls are essential for most analyses

Lecture Compilation Methods SS 2013 / Slide 229

Objectives:

Overview on oo analysis issues

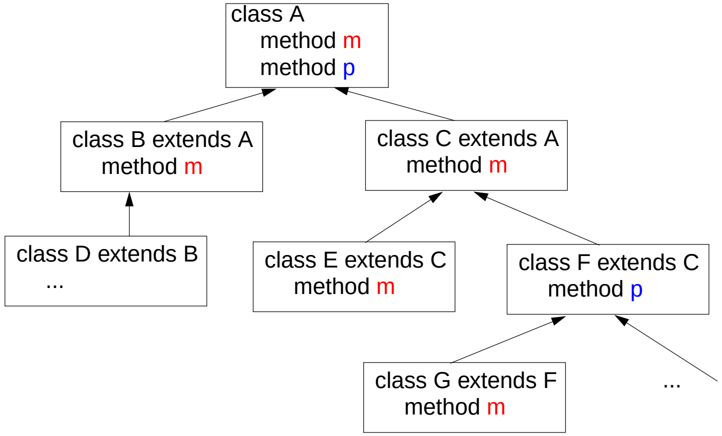
In the lecture:

- Role of class hierarchy for program analysis.
- Role of dynamic method binding for program analysis.

Class Hierarchy Graph

Node: **class or interface**

Arc a -> b: a is **subclass of** b or a **implements interface** b



Lecture Compilation Methods SS 2013 / Slide 230

Objectives:

Example for further consideration

In the lecture:

Recall central OO language properties:

- class hierarchy and typing,
- typed variables and method calls v.m(),
- inheritance of methods,
- overriding of methods,
- dynamically bound calls

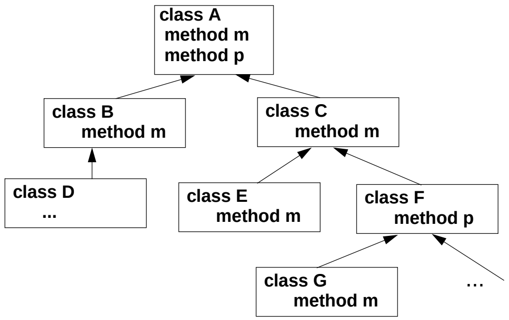
Assignments:

Recall the above mentioned language properties for Java and C++.

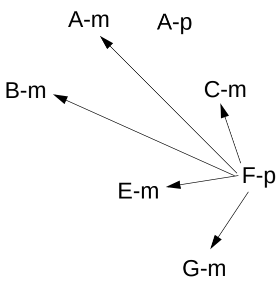
Object-Oriented Call Graph

Node: **implemented method**, identified by class name, method name: X-a

Arc X-a -> Y-b: method **X-a contains a call** v.b(...) that **may be bound to Y-b**



Call graph for F-p containing v.m(...)



Call graph: **any method m** may be bound to that call in F-p
(compare to function variables)
analysis yields better approximations

Lecture Compilation Methods SS 2013 / Slide 231

Objectives:

Understand the call graph problem

In the lecture:

The topics on the slide are explained. using the example.

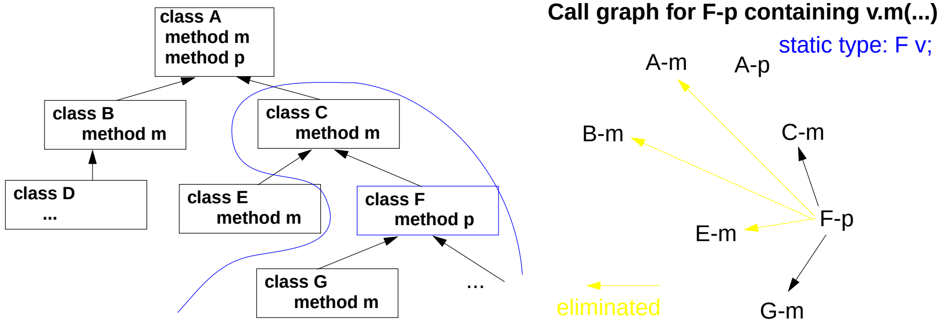
Call Graphs Constructed by Class Hierarchy Analysis (CHA)

The call graph is reduced to a set of **reachable methods** using the **class hierarchy** and the **static type of the receiver** expression in the call:

If a method F-p **is reachable** and
if it contains a **dynamically bound call v.m(...)** and
T is the static type of v,

then every method **m that is inherited by T or by a subtype of T is also reachable**, and arcs go from F-p to them.

Call graph for F-p containing v.m(...)



Lecture Compilation Methods SS 2013 / Slide 232

Objectives:

In the lecture:

The CHA method is explained using the example.

Refined Approximations for Call Graph Construction

Class Hierarchy Analysis (CHA): (see C-2.32)

Rapid Type Analysis (RTA):

As CHA, but only methods of those classes C are considered which are instantiated (`new C()`) in a reachable method.

Reaching Type Analysis:

Approximations of run-time types is propagated through a graph: nodes represent variables, arcs represent copy assignments.

Declared Type Analysis:
one node T represents all variables declared to have type T

Variable Type Analysis:
one node V represents a single variable

Points-to Analysis:

Information on object identities is propagated through the control-flow graph

Lecture Compilation Methods SS 2013 / Slide 233

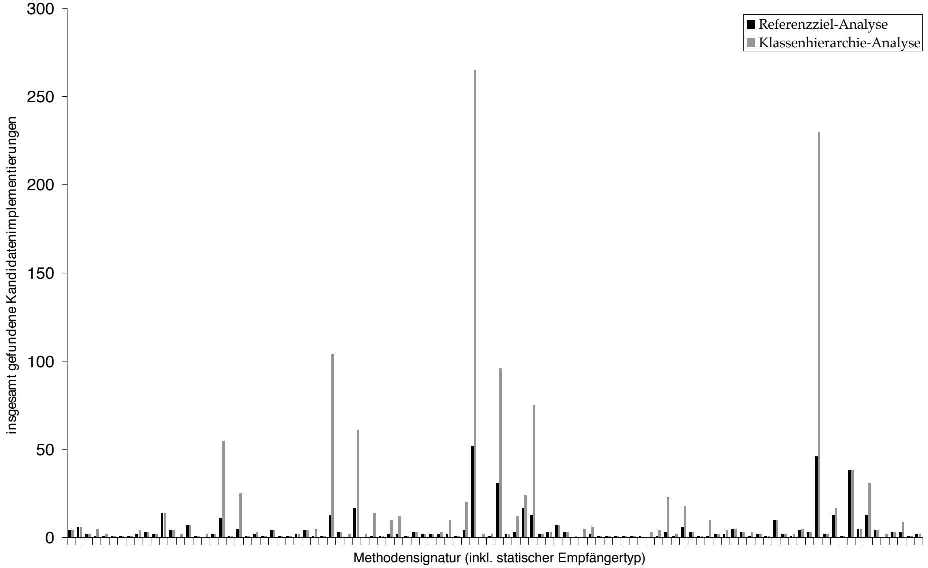
Objectives:

Powerful OO type analyses

In the lecture:

The methods are explained using small examples.

Results of Analysis of Dynamically Bound Calls



Lecture Compilation Methods SS 2013 / Slide 234

Objectives:

Effects on call identification

In the lecture:

The topics on the slide are explained. Examples are given.

- A pair of bars characterizes the number of method implementations, that may be bound to a set of calls having a particular type characteristics.
- Compare the results for CHA and points-to analysis.

Modules of a Toolset for Program Analysis

. .

analysis module	purpose	category
ClassMemberVisibility	examines visibility levels of declarations	visualization
MethodSizeStatistics	examines length of method implementations in bytecode operations and frequency of different bytecode operations	
ExternalEntities	histogram of references to program entities that reside outside a group of classes	
InheritanceBoundary	histogram of lowest superclass outside a group of classes	
SimpleSetterGetter	recognizes simple access methods with bytecode patterns	
MethodInspector	decomposes the raw bytecode array of a method implementation into a list of instruction objects	auxiliary analysis
ControlFlow	builds a control flow graph for method implementations	fundamental analyses
Dominator	constructs the dominator tree for a control flow graph	
Loop	uses the dominator tree to augment the control flow graph with loop and loop nesting information	
InstrDefUse	models operand accesses for each bytecode instruction	
LocalDefUse	builds intraprocedural def / use chains	
LifeSpan	analyzes lifeness of local variables and stack locations	
DefUseTypeInfo	infers type information for operand accesses	analysis of incomplete programs
Hierarchy	class hierarchy analysis based on a horizontal slice of the hierarchy	
PreciseCallGraph	builds call graph based on inferred type information, copes with incomplete class hierarchy	
ParamEscape	transitively traces propagation of actual parameters in a method call (escape = leaves analyzed library)	
ReadWriteFields	transitive liveness and access analysis for instance fields accessed by a method call	

Table 0-1. Analysis plug-ins in our framework

[Michael Thies: Combining Static Analysis of Java Libraries with Dynamic Optimization, Dissertation, Shaker Verlag, April 2001]

Lecture Compilation Methods SS 2013 / Slide 235

Objectives:

See analysis methods provided by a tool

In the lecture:

Some modules are related to methods presented in this lecture.

Questions:

Which modules implement a method that is presented in this lecture?

C-3.1

3. Code Generation

Input: Program in intermediate language

Tasks:

Storage mapping	properties of program objects (size, address) in the definition module
Code selection	generate instruction sequence, optimizing selection
Register allocation	use of registers for intermediate results and for variables

Output: abstract machine program, stored in a data structure

Design of code generation:

- analyze **properties of the target processor**
- plan **storage mapping**
- design at least one **instruction sequence** for each operation of the intermediate language

Implementation of code generation:

- Storage mapping:
  a traversal through the program and the definition module computes
  sizes and addresses of storage objects
- Code selection: use a generator for pattern matching in trees
- Register allocation:
  methods for expression trees, basic blocks, and for CFGs

Lecture Compilation Methods SS 2011 / Slide 301

Objectives:

Overview on design and implementation

In the lecture:

- Identify the 3 main tasks.
- Emphasize the role of design.

Suggested reading:

Kastens / Übersetzerbau, Section 7

C-3.2

3.1 Storage Mapping

Objective:
for each storable program object compute storage class, relative address, size

Implementation:
use properties in the definition module, traverse defined program objects

Design the use of storage areas:

code storage	progam code
global data	to be linked for all compilation units
run-time stack	activation records for function calls
heap	storage for dynamically allocated objects, garbage collection
registers for	addressing of storage areas (e. g. stack pointer) function results, arguments local variables, intermediate results (**register allocation**)

Design the mapping of data types (next slides)
Design activation records and translation of function calls (next section)

Lecture Compilation Methods SS 2011 / Slide 302

Objectives:

Design the mapping of the program state on to the machine state

In the lecture:

Explain storage classes and their use

Suggested reading:

Kastens / Übersetzerbau, Section 7.2

Storage Mapping for Data Types

Basic types

arithmetic, boolean, character types

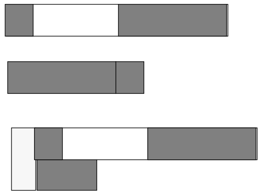
match language requirements and machine properties:
data format, available instructions,
size and alignment in memory

Structured types

for each type — representation in memory and code sequences for operations, e. g. assignment, selection, ...

record — relative address and alignment of components; reorder components for optimization

union — storage overlay, tag field for discriminated union



set — bit vectors, set operations

for **arrays** and **functions** see next slides

Lecture Compilation Methods SS 2011 / Slide 303

Objectives:

Overview on type mapping

In the lecture:

The topics on the slide are explained. Examples are given.

- Give examples for mapping of arithmetic types.
- Explain alignment of record fields.
- Explain overlay of union types.
- Discuss a recursive algorithm for type mapping that traverses type descriptions.

Suggested reading:

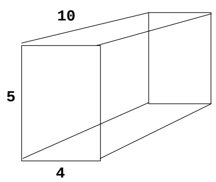
GdP slides on data types

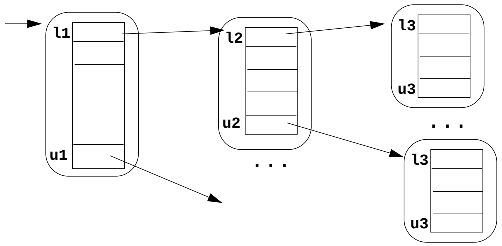
Array Implementation: Pointer Trees

An n-dimensional array

```
a: array[l1..u1, l2..u2, ..., ln..un] of real;
```

is implemented by a **tree of linear arrays**;
n-1 levels of pointer arrays and data arrays on the n-th level





Each single array can be allocated separately, dynamically; scattered in memory

In **Java arrays** are implemented this way.

Lecture Compilation Methods SS 2011 / Slide 304

Objectives:

Understand implementation variant

In the lecture:

Aspects of this implementation variant are explained:

- allocation by need,
- non-orthogonal arrays,
- additional storage for pointers,
- costly indirect access

Assignments:

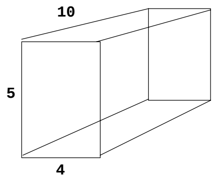
Allocate an array in Java that has the shape of a pyramid. How many pointer and data cells are needed?

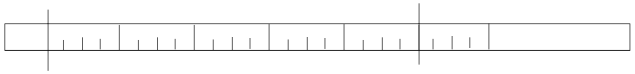
Array Implementation: Contiguous Storage

An n-dimensional array

```
a: array[l1..u1, l2..u2, ..., ln..un] of real;
```

is mapped to **one contiguous storage area linearized in row-major order**:





`start`
`store[start] ... store[start + elno*elsz - 1]`

linear storage map of array a onto byte-array **`store`** from index **`start`**:

number of elements	`elno = st1 * st2 * ... * stn`
i-th index stride	`sti = ui - li + 1`
element size in bytes	`elsz`

Index map of **`a[i1, i2, ..., in]`**:

```
store[start+ (..((i1-l1)*st2 + (i2-l2))*st3 +..)*stn + (in-ln))*elsz]

store[const + (..(i1*st2     + i2)*st3     +..)*stn +  in)*elsz]
```


Lecture Compilation Methods SS 2011 / Slide 305

Objectives:

Understand implementation variant

In the lecture:

Aspects of this implementation variant are explained:

- Give an example for a 3-dimensional array.
- Explain the index function.
- Explain the index function with constant terms extracted.
- Compare the two array implementation variants:
- Allocation in one chunk,
- orthogonal arrays only,
- storage only for data elements,
- efficient direct addressing.
- FORTRAN: column major order!

Suggested reading:

GdP slides on data types

Questions:

- What information is needed in an array descriptor for a dynamically allocated multi-dimensional array?

Functions as Data Objects

Functions may occur **as data objects**:

- variables
- parameters
- function results
- lambda expressions (in functional languages)

Functions that are defined on the **outermost program level** (non-nested)

can be implemented by just the **address of the code**.

Functions that are **defined in nested structures** have to be implemented by a **pair: (closure, code)**

The **closure** contains all **bindings** of names to variables or values that are valid when the **function definition is executed**.

In **run-time stack** implementations the **closure is a sequence of activation records on the static predecessor chain.**

Lecture Compilation Methods SS 2011 / Slide 306

Objectives:

Understand the concept of closure

In the lecture:

The topics on the slide are explained:

- examples for functions as data objects,
- recall functional programming (GdP),
- closures as a sequence of activation records,
- relate closures to run-time stacks

Suggested reading:

GdP slides on run-time stack

Questions:

- Why must a functional parameter in Pascal be represented by a pair (closure, code)?

3.2 Run-Time Stack
Activation Records

Run-time stack contains one **activation record** for each active function call.

Activation record:
provides storage for the data of a function call.

dynamic link:
link from callee to caller,
to the preceding record on the stack

static link:
link from callee c to the record s where c is defined

s is a call of a function which contains the definition
of the function, the call of which created c.

Variables of surrounding functions are
accessed via the static predecessor chain.

Only relevant for languages which allow
nested functions, classes, objects.

closure of a function call:
the **activation records on the static predecessor chain**

activation record:

parameters
static link
return address
dynamic link
local variables
register save area

Objectives:
Understand activation records

In the lecture:
Explain

- static and dynamic links,
- Explain nesting and closures,
- return address.

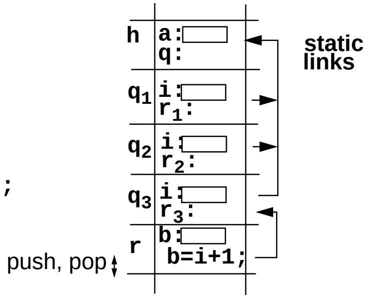
See C-3.10 for relation to call code.

Example for a Run-Time Stack

Run-time stack:
A call creates an activation record and pushes it onto the stack.
It is popped on termination of the call.

nested functions

```
h  float a;
   q  int i;
      r  int b;
         b=i+1;
      if(..) q();
      r();
   q();
```



The **static link** points to the activation record where the called function is defined, e. g. r_3 in q_3

Optimization: activation records of **non-recursive functions** may be allocated statically.
Languages without recursive functions (FORTRAN) do not need a run-time stack.

Parallel processes, threads, and coroutines need a **separate run-time stack** each.

Objectives:
Understand run-time stacks

In the lecture:

- Explain static links.
- Explain nesting and closures.

Questions:

- Why do threads need a separate run-time stack?

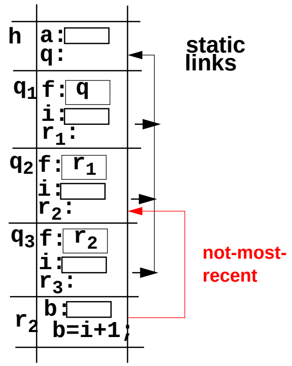
Not-Most-Recent Property

The **static link** of an activation record c for a function r
points to an activation record d for a function q where r is defined in.
If there are activation records for q on the stack, that are more recently created than d,
the **static link to d is not-most-recent**.

That effect can be achieved by using functional parameters or variables.
Example:

nested functions

```
h  float a;

   q(funct f)
      int i;

      r  int b;
         b=i+1;

      if(..) q(r);

      *f();

   q(q);
```



Lecture Compilation Methods SS 2011 / Slide 309

Objectives:

Really understand static links

In the lecture:

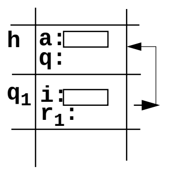
- Explain not-most-recent property.
- r[1] and r[2] must be represented by different values, because they have different closures.

Closures on Run-Time Stacks

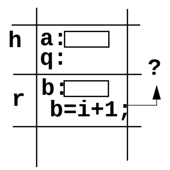
Function calls can be implemented by a run-time stack if the
closure of a function is still on the run-time stack when the function is called.

```
h  float a;

   q  int i;

      r  int b;
         b=i+1;

      return r;

   *(q()) ();
```

Example for violation:



during the call of q



the closure for the call of r is missing

Language conditions to guarantee run-time stack discipline:

Pascal:	functions not allowed as function results, or variables
C:	no nested functions
Modula-2:	nested functions not allowed as values of variables

Functional languages maintain activation records on the heap instead of the run-time stack

Lecture Compilation Methods SS 2011 / Slide 310

Objectives:

Language condition for run-time stacks

In the lecture:

- Explain language restrictions to ensure that necessary closures are on the run-time stack.

Questions:

- Explain why C, Pascal, and Modula-2 obey the requirement on stack discipline?

Activation Records and Call Code

activation record:

- result
- parameters
- static link
- return address
- dynamic link
- local variables
- register save area

\- ↑ +

← 0
base
address

\+ ↓ -

call code

push parameter values
push static link
subroutine jump

function code

push dynamic link
stack register := top of stack
increment top of stack
for local variables
save registers
...
function body
...
restore registers
deallocate local variables
pop stack register
return jump

call code

pop static link
pop parameter area
use and pop result

Lecture Compilation Methods SS 2011 / Slide 311

Objectives:

Relation between activation record and call code

In the lecture:

Explain

- contents of records,
- how to save registers,
- relative addresses of data in the activation record
- register windowing related to run-time stacks

Suggested reading:

Kastens / Übersetzerbau, Section 7.2.2, 7.3.1

Questions:

- How would you design the layout of activation records for a processor that provides register windowing?

3.3 Code Sequences for Control Statements

A **code sequence** defines how a **control statement** is transformed into jumps and labels.

Notation of the **`Code`** constructs:

`Code (S)`	generate code for statements **`S`**
`Code (C, true, M)`	generate code for condition **`C`** such that it branches to M if **`C`** is true, otherwise control continues without branching
`Code (A, Ri)`	generate code for expression **`A`** such that the result is in register **`Ri`**

Code sequence for if-else statement:

```
if (cond) ST; else SE;:

        Code (cond, false, M1)
        Code (ST)
        goto M2
   M1:  Code (SE)
   M2:
```


Lecture Compilation Methods SS 2011 / Slide 312

Objectives:

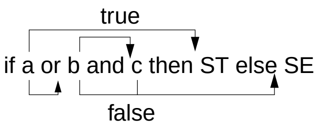
Concept of code sequences for control structures

In the lecture:

- Explain the notation.
- Explain the code sequence for if-else statements.

Short Circuit Translation of Boolean Expressions

Boolean expressions are translated into **sequences of conditional branches**.
Operands are evaluated from left to right until the result is determined.



2 code sequences for each operator; applied to condition tree on a top-down traversal:

Code (A and B, true, M):	Code (A, false, N) Code (B, true, M) N:
Code (A and B, false, M):	Code (A, false, M) Code (B, false, M)
Code (A or B, true, M):	Code (A, true, M) Code (B, true M)
Code (A or B, false, M):	Code (A, true, N) Code (B, false, M) N:

Code (not A, X, M):	Code (A, not X, M)
Code (A < B, true, M):	Code (A, Ri); Code (B, Rj) cmp Ri, Rj braLt M
Code (A < B, false, M):	Code (A, Ri); Code (B, Rj) cmp Ri, Rj braGe M
Code for a leaf:	conditional jump

Lecture Compilation Methods SS 2011 / Slide 313

Objectives:

Special technique for translation of conditions

In the lecture:

- Explain the transformation of conditions.
- Use the example of C-3.14
- Use 2 inherited attributes for the target label and the case when to branch.
- Discuss whether the technique may be applied for C, Pascal, and Ada.

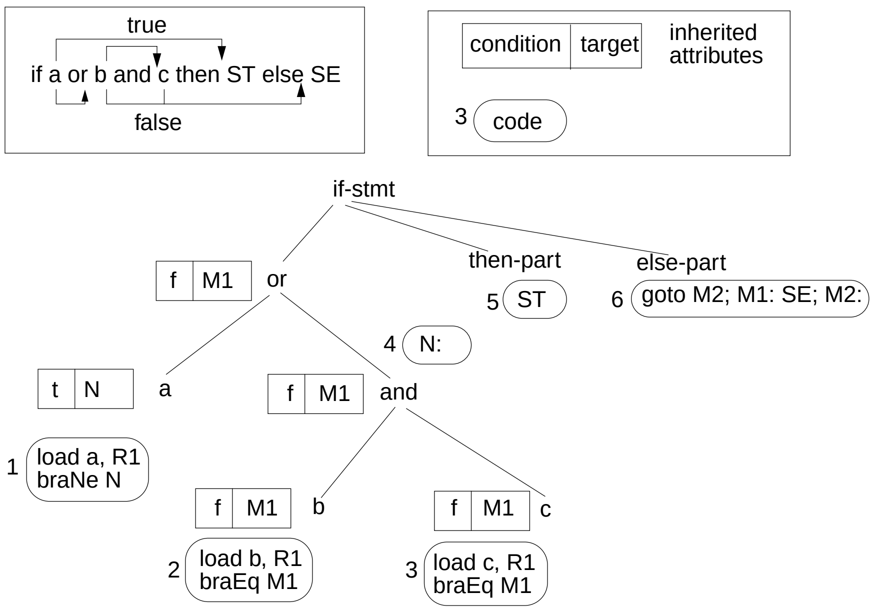
Suggested reading:

Kastens / Übersetzerbau, Section 7.3.3

Questions:

- Why does the transformation of conditions reduce code size?
- How is the technique described by an attribute grammar?
- Why is no instruction generated for the operator *not* ?
- Discuss whether the technique may or must be applied for C, Pascal, and Ada.

Example for Short Circuit Translation



Lecture Compilation Methods SS 2011 / Slide 314

Objectives:

Illustrate short circuit translation

In the lecture:

Discuss together with C-3.13

Suggested reading:

Kastens / Übersetzerbau, Section 7.3.3

Code Sequences for Loops

While-loop variant 1:

```
while (Condition) Body

   M1:   Code (Condition, false, M2)
         Code (Body)
         goto M1
   M2:
```

While-loop variant 2:

```
while (Condition) Body

         goto M2
   M1:   Code (Body)
   M2:   Code (Condition, true, M1)
```

Pascal for-loop unsafe variant:

```
for i:= Init to Final do Body

      i = Init
   L: if (i>Final) goto M
      Code (Body)
      i++
      goto L
   M:
```

Pascal for-loop safe variant:

```
for i:= Init to Final do Body

      if (Init==minint) goto L
      i = Init - 1
      goto N
   L: Code (Body)
   N: if (i>= Final) goto M
      i++
      goto L
   M:
```


Lecture Compilation Methods SS 2011 / Slide 315

Objectives:

Understand loop code

In the lecture:

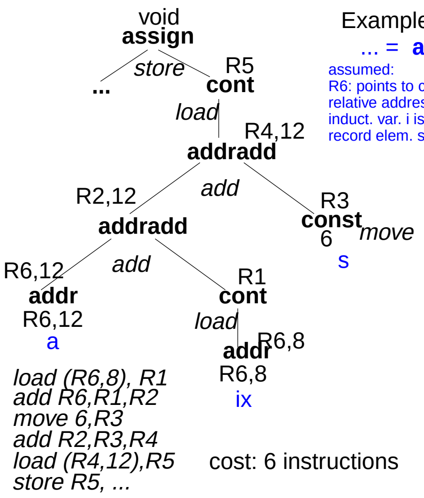
- Explain the code sequences for while-loops.
- Discuss the two variants.
- Explain the code sequences for for-loops.
- Variant 1 may cause an exception if Final evaluates to maxint.
- Variant 2 avoids that problem.
- Variant 2 needs further checks to avoid an exception if Init evaluates to minint.
- Both variants should not evaluate the Final expression on every iteration.

Questions:

- What are the advantages or problems of each alternative?

3.4 Code Selection

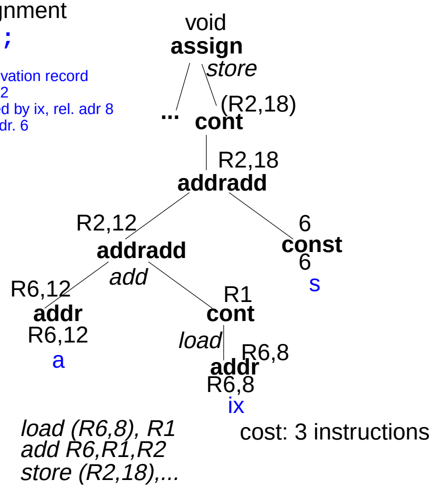
- Given: target tree in intermediate language.
- **Optimizing selection: Select patterns** that translate single nodes or small subtrees into machine instructions; cover the whole tree with as few instructions as possible.
- Method: **Tree pattern matching,** several techniques



Example: assignment

... = a[i].s;

assumed:
R6: points to current activation record
relative address of a is 12
induct. var. i is substituted by ix, rel. adr 8
record elem. s has rel. adr. 6



Lecture Compilation Methods SS 2011 / Slide 316

Objectives:

Understand the task

In the lecture:

The topics on the slide are explained. Examples are given.

- The task is explained.
- Example: Code of different cost for the same tree.

Selection Technique: Value Descriptors

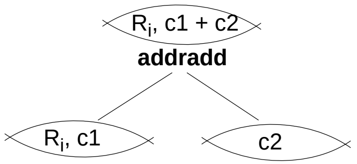
Intermediate language **tree node operators**; e.g.:

addr	address of variable
const	constant value
cont	load contents of address
addradd	address + value

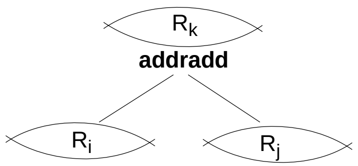
Value descriptors state how/where the value of a tree node is represented, e. g.

$\mathbf{R_i}$	value in register R_i
c	constant value c
$\mathbf{R_i,c}$	address R_i + c
(adr)	contents at the address adr

alternative **translation patterns** to be selected context dependend:



addradd R_i, c1 c2 -> R_i, c1 + c2 ./.



addradd R_i R_j -> R_k add R_i, R_j, R_k

Lecture Compilation Methods SS 2011 / Slide 317

Objectives:

Notion of value descriptors

In the lecture:

- Explain value descriptors
- Explain alternative translation patterns
- Concept of deferred operations
- Different costs of translations
- Compare with the concept of overloaded operators: here, selection by kind of value descriptor.

Suggested reading:

Kastens / Übersetzerbau, Section 7.3.4

Questions:

- How is the technique related to overloaded operators in source languages?

Example for a Set of Translation Patterns

#	operator	operands		result	code
1	**addr**	R_i, c		-> R_i,c	./.
2	**const**	c		-> c	./.
3	**const**	c		-> R_i	move c, R_i
4	**cont**	R_i, c		-> (R_i, c)	./.
5	**cont**	R_i		-> (R_i)	./.
6	**cont**	R_i, c		-> R_j	load (R_i, c), R_j
7	**cont**	R_i		-> R_j	load (R_i), R_j
8	**addradd**	R_i	c	-> R_i, c	./.
9	**addradd**	R_i, c1	c2	-> R_i, c1 + c2	./.
10	**addradd**	R_i	R_j	-> R_k	add Ri, R_j, R_k
11	**addradd**	R_i, c	R_j	-> R_k, c	add R_i, R_j, R_k
12	**assign**	R_i	R_j	-> void	store R_j, R_i
13	**assign**	R_i	(R_j, c)	-> void	store (R_j,c), R_i
14	**assign**	R_i,c	R_j	-> void	store R_j, R_i,c

Lecture Compilation Methods SS 2011 / Slide 318

Objectives:

Example

In the lecture:

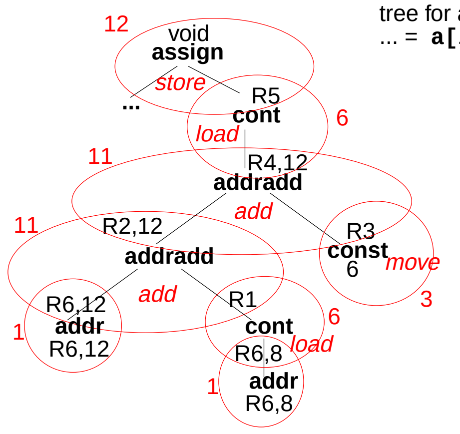
- Explain the meaning of the patterns.
- Use the example for the tree of C-3.19

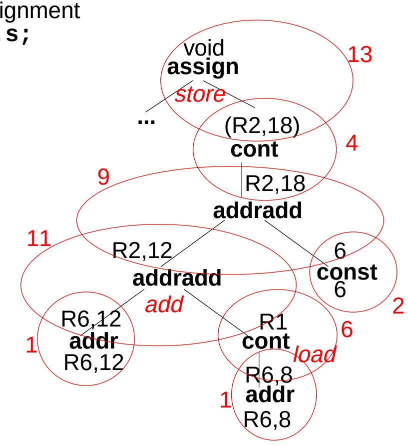
Suggested reading:

Kastens / Übersetzerbau, Section 7.3.4

Tree Covered with Translation Patterns

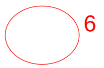
tree for assignment
... = `a[i].s;`





load (R6,8), R1
add R6,R1,R2
move 6,R3
add R2,R3,R4
load (R4,12),R5
store R5, ...

cost: 6 instructions



application of pattern #6

load (R6,8), R1
add R6,R1,R2
store (R2,18),...

cost: 3 instructions

Lecture Compilation Methods SS 2011 / Slide 319

Objectives:

Example for pattern applications

In the lecture:

- Show applications of patterns.
- Show alternatives and differences.
- Explain costs accumulated for subtrees.
- Compose code in execution order.

Pattern Selection

Pass 1 bottom-up:

Annotate the nodes with sets of pairs
{ (v, c) | v is a kind of value descriptor that an applicable pattern yields,
c are the accumulated subtree costs}

If (v, c1), (v, c2) keep only the cheaper pair.

Pass 2 top-down:

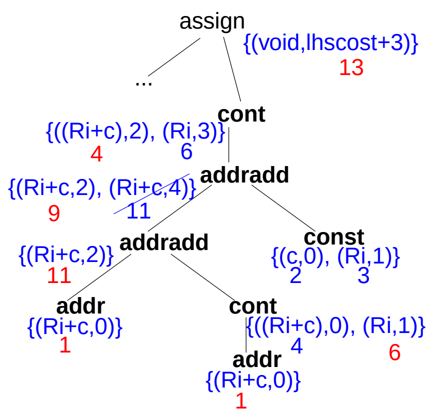
Select for each node the cheapest pattern,
that fits to the selection made above.

Pass 3 bottom-up:

Emit code.

Improved technique:

relative costs per sets =>
finite number of potential sets
integer encoding of the sets at generation time



load (R6,8), R1
add R6,R1,R2
store (R2,18),...

cost: 3 instructions

Lecture Compilation Methods SS 2011 / Slide 320

Objectives:

2-pass selection algorithm

In the lecture:

- Explain the role of the pairs and sets.
- Show the selection using the following pdf file: an example for pattern selection
- Overloading resolution in Ada is performed in a similar way (without costs).

Pattern Matching in Trees: Bottom-up Rewrite

Bottom-up Rewrite Systems (BURS) :
a general approach of the pattern matching method:

Specification in form of tree patterns, similar to C-3.18 - C-3.20

Set of patterns is **analyzed at generation** time.

Generator produces a **tree automaton** with a finite set of states.

On the bottom-up traversal it annotates each tree node with
a **set of states**:
those selection decisions which may lead to an optimal solution.

Decisions are made on the base of the **costs of subtrees**
rather than costs of nodes.

Generator: BURG

Lecture Compilation Methods SS 2011 / Slide 321

Objectives:

Get an idea of the BURS method

In the lecture:

- Explain the basic ideas of BURS.
- Compare it to the previous technique.
- Decides on the base of subtree costs.
- Very many similar patterns are needed.

Suggested reading:

Kastens / Übersetzerbau, Section 7.4.3

Questions:

- In what sense must the specification be complete?

Tree Pattern Matching by Parsing

The tree is represented in prefix form.

Translation patterns are specified by tuples (CFG production, code, cost),
Value descriptors are the nonterminals of the grammar, e. g.

8	RegConst ::= **addradd** Reg Const	nop	0
11	RegConst ::= **addradd** RegConst Reg	add R_i, R_j, R_k	1

Deeper patterns allow for more effective optimization:

Void ::= **assign** RegConst **addradd** Reg Const	store (Ri, c1),(Rj, c2)	1

Parsing for an ambiguous CFG:
application of a production is decided on the base of the production costs
rather than the accumulated subtree costs!

Technique „Graham, Glanville“
Generators: GG, GGSS

Lecture Compilation Methods SS 2011 / Slide 322

Objectives:

Understand the parsing approach

In the lecture:

Explain

- how a parser performs a tree matching,
- that the parser decides on the base of production costs,
- that the grammar must be complete,
- that very many similar patterns are needed.

Suggested reading:

Kastens / Übersetzerbau, Section 7.4.3

Questions:

- In what sense must the grammar be complete? What happens if it is not?
- Why is it desirable that the grammar is ambiguous?
- Why is BURS optimization more effective?

4 Register Allocation

Use of registers:

1. intermediate **results of expression evaluation**
2. reused results of expression evaluation (CSE)
3. contents of frequently used **variables**
4. **parameters** of functions, **function result**
   (cf. register windowing)
5. stack pointer, **frame pointer**, heap pointer, ...

Number of registers is limited - for each register class: address, integer, floating point

Specific allocation methods for different context ranges:

- 4.1 expression trees (Sethi, Ullman)
- 4.2 basic blocks (Belady)
- 4.3 control flow graphs (graph coloring)

Register allocation aims at reduction of

- number of memory accesses
- spill code, i. e. instructions that store and reload the contents of registers

Symbolic registers: allocate a new symbolic register to each value assignment (single assignment, no re-writing); defer allocation of real registers to a later phase.

Lecture Compilation Methods SS 2013 / Slide 401

Objectives:

Overview on register allocation

In the lecture:

Explain the use of registers for different purposes.

Suggested reading:

Kastens / Übersetzerbau, Section 7.5

Register Windowing

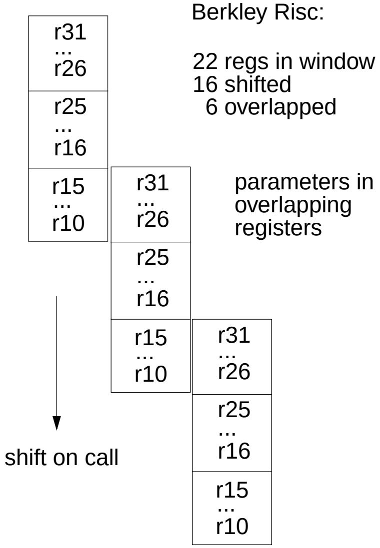
Register windowing:

- Fast storage of the processor is accessed through a window.
- The n elements of the window are used as registers in instructions.
- On a call the window is shifted by m<n registers.
- Overlapping registers can be used under different names from both the caller and the callee.
- Parameters are passed without copying.
- Storage is organized in a ring;
  4-8 windows; saved and restored as needed

Typical for Risc processors,
e.g. Berkley RISC, SPARC

Berkley Risc:

22 regs in window
16 shifted
6 overlapped



Lecture Compilation Methods SS 2013 / Slide 402

Objectives:

Understand the technique of register windowing

In the lecture:

Explain the technique.

Suggested reading:

Kastens / Übersetzerbau, Section 7.5

Suggested reading:

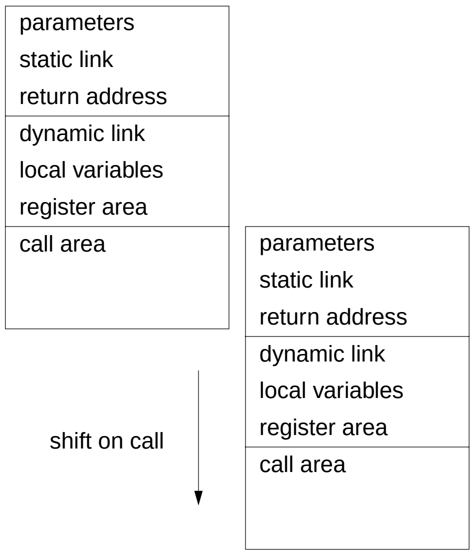
Lecture "Grundlagen der Rechnerarchitektur"

Questions:

- Describe a situation when large runtime costs are caused by save and restore of the ring storage.

Activation Records in Register Windows

- **Parameters** are passed in overlap area **without copying**.
- **Registers need not be saved** explicitly.
- If **window is too small** for an activation record, the remainder is allocated on the **run-time stack**; pointer to it in window.



Lecture Compilation Methods SS 2013 / Slide 403

Objectives:

Use of register windowing

In the lecture:

- Explain how the technique is used.
- Explain the relation to the run-time stack.

Suggested reading:

Kastens / Übersetzerbau, Section 7.5

Questions:

- Under what restriction can the register windows completely substitute the activation records of certain functions?

4.1 Register Allocation for Expression Trees

Problem:

Generate code for **expression** evaluation.
Intermediate results are stored in registers.
Not enough registers:
spill code saves and restores.

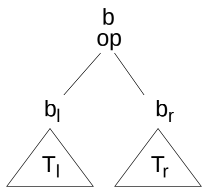
Goal:

Minimize amount of spillcode.
see C-4.5a for optimality condition

Basic idea (Sethi, Ullman):

For each subtree minimize the **number of needed registes**:

evaluate **first the subtree that needs most** registers



assume the results of T_l and T_r are in registers

eval. order	needed registers b =	
T_l T_r op	max (b_l, b_r + 1)	minimize
T_r T_l op	max (b_r, b_l + 1)	

number of available registers (regmax) is upper limit for needed registers

Lecture Compilation Methods SS 2013 / Slide 404

Objectives:

Select evaluation order determines number of needed registers

In the lecture:

- Show that evaluation order determines the number of registers needed for a subtree.
- Explain the computation of needed registers.

Suggested reading:

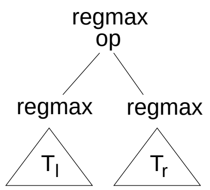
Kastens / Übersetzerbau, Section 7.5.3

Assignments:

- Apply the technique for several register classes.

Expression Tree Attribution

Spill code needed:

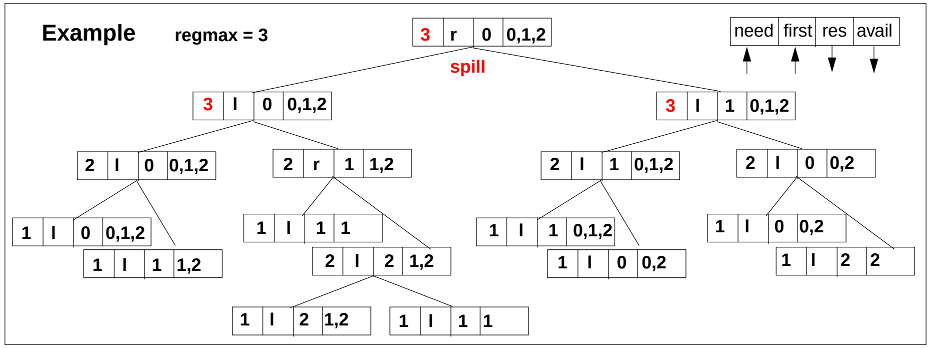


Code (T_r)
store R_r, h
Code (T_l)
load h, R_r
op R_r, R_l

load h, R_r is not needed if h can be a memory operand in op h, R_l

Implementation by attribution of trees:

- **Phase 1** bottom-up: needed registers, evaluation order
- **Phase 2** top-down: allocate registers
- **Phase 3** bottom-up: compose code in evaluation order



Lecture Compilation Methods SS 2013 / Slide 405

Objectives:

Tree attribution in phases

In the lecture:

- Explain the spill code situation.
- Explain the example.
- Explain in attribute grammar terminology.

Suggested reading:

Kastens / Übersetzerbau, Section 7.5.3

Questions:

- Assume that in an expression tree spill code is generated at 2 nodes. Where are these nodes?
- Specify the technique by an attribute grammar.

Contiguous code vs. optimal code

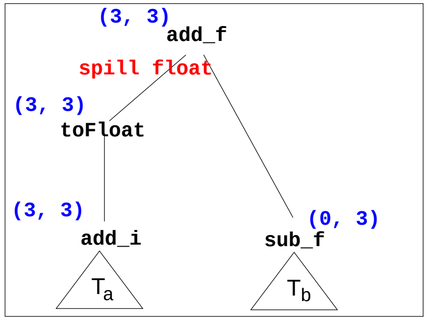
The method assumes that the **code for every subtree is contiguous**.
(I.e. there is no interleaving between the code of any two disjoint subtrees.)

The **method is optimal** for a certain **configuration of registers and operations**, iff every **optimal evaluation code** can be arranged to be **contiguous**.

Counter example:

Registers: 3 `int` and 3 `float`
Register need: (i, f) from (0, 0) to (3, 3)

Operations: `int`- and `float`- arithmetic, `toFloat` (widening)



register use:	(3, 3)	(1, 0)	(0, 1)	(0, 0) (0, 3)	(0, 1)	(0, 2)	(0, 1)		
contiguous:	T_a	`add_i`	`toFloat`	`store_f`	T_b	`sub_f`	`load_f`	`add_f`	
optimal:	T_a	`add_i`	T_b	`sub_f`	`toFloat`	`add_f`			
register use:	(3, 3)	(1, 0)	(1, 3)	(1, 1)	(1, 2)	(0, 1)			

Lecture Compilation Methods SS 2013 / Slide 405a

Objectives:

Understand the optimality condition

In the lecture:

- Explain the condition for optimality.
- Explain the counter example.

Suggested reading:

Kastens / Übersetzerbau, Section 7.5.3

C-4.6

4.2 Register Allocation for Basic Blocks by Life-Time Analysis

Lifetimes of values in a basic block are used to minimize the number of registers needed.

1st Pass: Determine the **life-times** of values: from the definition to the last use (there may be several uses!).

Life-times are represented by intervals in a graph

cut of the graph = number of **registers needed** at that point

at the end of 1st pass:
maximal cut = number of register needed for the basic block

allocate registers **in the graph**:

In case of shortage of registers: select values to be **spilled**; **criteria**:

- a **value that is already in memory** - store instruction is saved

- the **value that is latest used again**

2nd Pass: allocate registers **in the instructions**; evaluation order remains unchanged

The technique has been presented originally 1966 by **Belady** as a **paging technique for storage allocation**.

Lecture Compilation Methods SS 2013 / Slide 406

Objectives:
Specify life-time and register need by interval graphs

In the lecture:
- Explain the technique using the example of C-4.7; show its characteristics:
- reused intermediate results,
- evaluation order remains unchanged,
- interpretation as a paging technique.

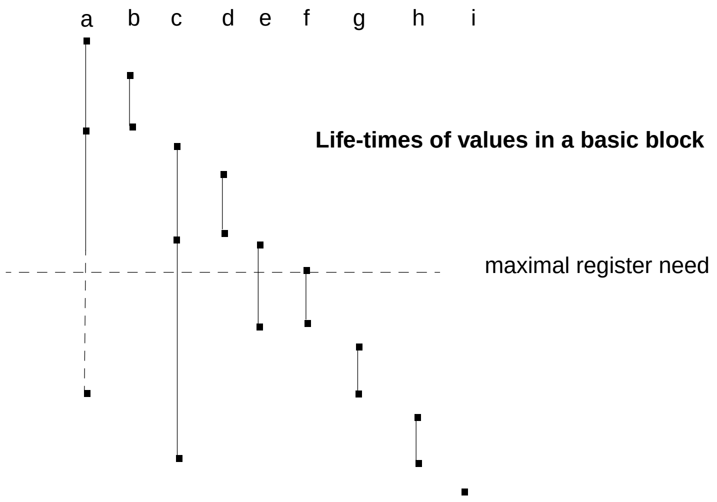
Suggested reading:
Kastens / Übersetzerbau, Section 7.5.2

Questions:
- Explain the criteria for selecting values to be spilled.
- Explain the technique in terms of memory paging.

C-4.7

Example for Belady‘s Technique

a := x
b := y
c := b + a
d := z
e := d * c
f := s
g := e / f
h := g + a
i := h * c



register allocations

4 regs	(a)	d1	d2	d2	d3	d3	d4	d3	d3	d3	
3 regs	(b)	d1	d2	d2*	d3	d3	d2	d3	d3	d3	* spilled: store;...; load
3 regs	(c)	d1*	d2	d2	d3	d3	d1	d3	d3	d3	* spilled: reloaded from x

Lecture Compilation Methods SS 2013 / Slide 407

Objectives:
Example for C-4.6

In the lecture:
Explain
- the example,
- the variants of allocation,
- the application of the selection criteria.

Suggested reading:
Kastens / Übersetzerbau, Section 7.5.2, Abb. 7.5-3

Assignments:
- Apply the technique for another example.

Questions:
- Explain the alternatives (b) and (c).

4.3 Register Allocation by Graph Coloring

Definitions and uses of variables in control-flow graphs for **function bodies** are analyzed (DFA).
Conflicting life-times are modelled. Presented by **Chaitin**.

Construct an interference graph:

- **Nodes:** Variables that are candidates for being kept in registers
- **Edge {a, b}:** **Life-times** of variables a and b overlap
  => a, b have to be kept in different registers

Life-times for CFGs are determined by **data-flow analysis**.

Graph is „colored“ with register numbers.

NP complete problem; **heuristic technique** for coloring with k colors (registers):

eliminate nodes of degree < k (and its edges)

if the graph is finally empty:
- graph can be colored with k colors
- assign colors to nodes in reverse order of elimination

else
- graph can not be colored this way
- select a node for spilling
- repeat the algorithm without that node

Lecture Compilation Methods SS 2013 / Slide 408

Objectives:

Overlapping life-times modelled by interference graphs

In the lecture:

- Explain the interference graph using the example of C-4.9.
- Demonstrate the heuristics.

Suggested reading:

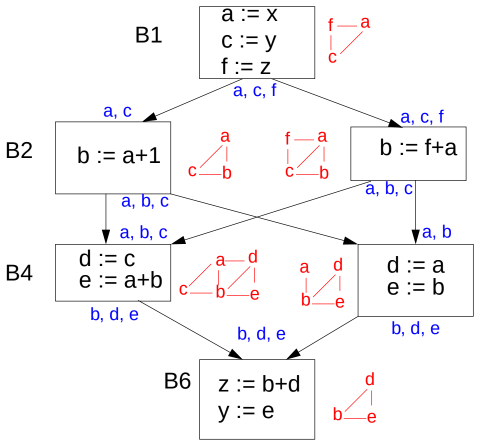
Kastens / Übersetzerbau, Section 7.5.4

Questions:

- Why is DFA necessary to determine overlapping life-times? Why can’t one check each block separately? Give an example where that simplified approach would yield wrong results.
- Show a graph that is k-colorable that is not colored successfully by this heuristic.

Example for Graph Coloring

CFG with definitions and uses of variables

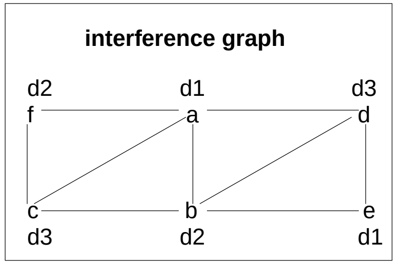


variables in memory: x, y, z

variables considered for register alloc.:
a, b, c, d, e, f

results of live variable analysis:
b, d, e

contribution to interference graph



Lecture Compilation Methods SS 2013 / Slide 409

Objectives:

Example for C-4.8

In the lecture:

Explain the example

Suggested reading:

Kastens / Übersetzerbau, Section 7.5.4, Fig. 7.5-6

Assignments:

- Apply the technique for another example.

Questions:

- Why is variable b in block B5 alive?

5 Code Parallelization

Processor with **instruction level parallelism (ILP)** executes several instructions in parallel.

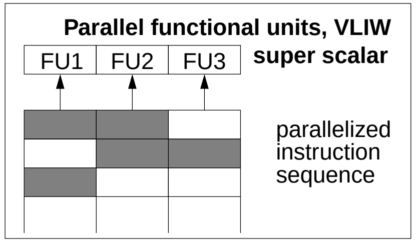
Classes of processors and parallelism:
VLIW, super scalar
Pipelined processors
Data parallel processors

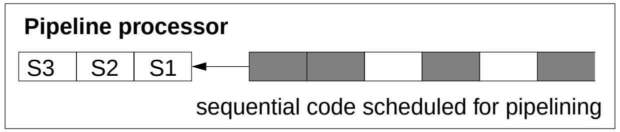
Compiler **analyzes sequential programs** to **exhibit potential parallelism** on instruction level;

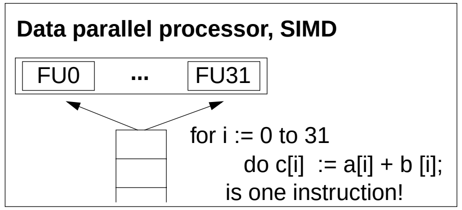
model **dependences between computations**

Compiler arranges instructions for shortest execution time: **instruction scheduling**

Compiler **analyzes loops** to execute them in parallel
loop transformation
array transformation







Lecture Compilation Methods SS 2013 / Slide 501

Objectives:

3 abstractions of processor parallism

In the lecture:

- explain the abstract models
- relate to real processors
- explain the instruction scheduling tasks

Suggested reading:

Kastens / Übersetzerbau, Section 8.5

Questions:

- What has to be known about instruction execution in order to solve the instruction scheduling problem in the compiler?

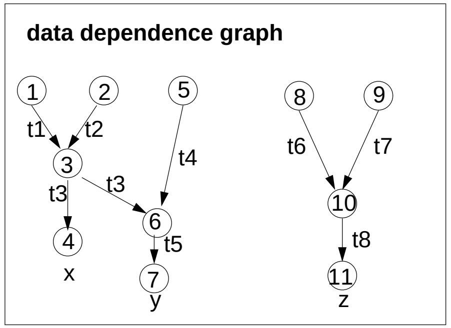
5.1 Instruction Scheduling Data Dependence Graph

Exhibit potential **fine-grained parallelism** among operations.
Sequential code is over-specified!

Data dependence graph (DDG) for a basic block:
Node: operation;
Edge a -> b: operation b uses the result of operation a

Example for a basic block:

1:	t1	:= a
2:	t2	:= b
3:	t3	:= t1 + t2
4:	x	:= t3
5:	t4	:= c
6:	t5	:= t3 + t4
7:	y	:= t5
8:	t6	:= d
9:	t7	:= e
10:	t8	:= t6 + t7
11:	z	:= t8



ti are symbolic registers, store intermediate results, obey single assignment rule

Lecture Compilation Methods SS 2013 / Slide 502

Objectives:

DDG exhibits parallelism

In the lecture:

- Show where sequential code is overspecified.
- Derive reordered sequences from the ddg.
- single assignment for ti: ti contains exactly one value; ti is not reused for other values.
- Without that assumption further dependencies have to manifest the order of assignments to those registers.

Suggested reading:

Kastens / Übersetzerbau, Section 8.5, Abb. 8.5-1

Assignments:

- Write the operations of the basic block in a different order, such that the effect is not changed and the same DDG is produced.

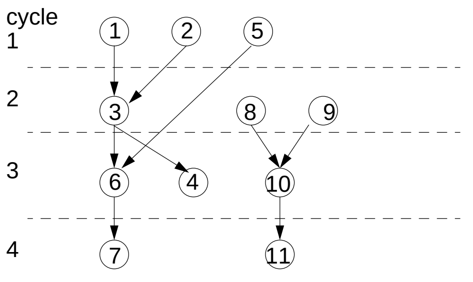
Questions:

- Why does this example have so much freedom for rearranging operations?
- Why are further dependences necessary if registers are allocated?

List Scheduling

Input: data dependence graph
Output: a schedule of **at most k operations per cycle**,
such that all **dependences point forward;** DDG arranged in levels

Algorithm: A **ready list** contains all operations that are **not yet scheduled**,
but whose **predecessors are scheduled**
Iterate: **select** from the ready list up to k operations for the next cycle (heuristic),
update the ready list



- Algorithm is **optimal** only for **trees**.
- **Heuristic**: Keep ready list sorted by distance to an end node, e. g.

  (1 2 5) (8 9 3) (6 10 4) (7 11)

  without this heuristic:
  (1 8 9) (2 5 10) (3 11) (6 4) (7)

  ( ) operations in one cycle

Critical paths determine minimal schedule length: e. g. 1 -> 3 -> 6 -> 7

Lecture Compilation Methods SS 2013 / Slide 503

Objectives:
A simple fundamental scheduling algorithm

In the lecture:
- Explain the algorithm using the example.
- Show variants of orders in the ready list, and their consequences.
- Explain the heuristic.

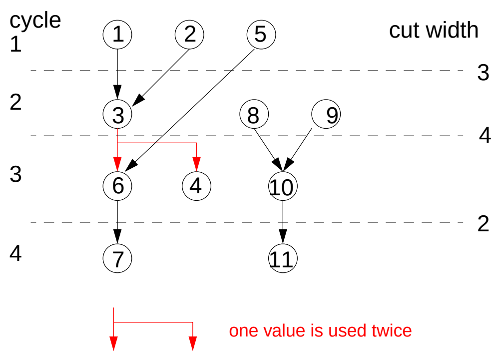
Suggested reading:
Kastens / Übersetzerbau, Section 8.5.1

Assignments:
- Write the parallel code for this example.

Questions:
- Explain the heuristic with respect to critical paths.

Variants and Restrictions for List Scheduling

- Allocate **as soon as possible**, ASAP (C-5.3); as **late** as possible, ALAP
- Operations have **unit execution time** (C-5.3); **different execution times:** selection avoids conflicts with already allocated operations
- Operations only on **specific functional units** (e. g. 2 int FUs, 2 float FUs)
- **Resource restrictions** between operations, e. g. <= 1 load or store per cycle



Scheduled DDG models **number of needed registers**:
- arc represents the use of an intermediate result
- **cut width** through a level gives the number of **registers needed**

The tighter the schedule the more registers are needed (*register pressure*).

Lecture Compilation Methods SS 2013 / Slide 504

Objectives:
A simple fundamental scheduling algorithm

In the lecture:
- Explain ASAP and ALAP.
- Explain restrictions on the selection of operations.
- Show how the register need is modeled.

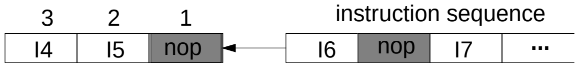
Suggested reading:
Kastens / Übersetzerbau, Section 8.5.1

Assignments:
- The algorithm allocates an operation as soon as possible (ASAP). Describe a variant of the algorithm which allocates an operation as late as possible (ALAP).
- Describe a variant, that allocates operations of different execution times.

Questions:
- Compare the way register need is modeled with the approach of Belady for register allocation.
- Why need tight schedules more registers?

Instruction Scheduling for Pipelining

Instruction pipeline with 3 stages:



Dependent instructions may not follow one another immediately.

Schedule rearranges the operation sequence, to minimize the number of delays:

without scheduling:

```
 1:   t1    := a
 2:   t2    := b
      nop
 3:   t3    := t1 + t2
      nop
 4:   x     := t3
 5:   t4    := c
      nop
 6:   t5    := t3 + t4
      nop
 7:   y     := t5
 8:   t6    := d
 9:   t7    := e
      nop
10:   t8    := t6 + t7
      nop
11:   z     := t8
```

```
 1:   t1    := a
 2:   t2    := b
 5:   t4    := c
 3:   t3    := t1 + t2      with
 8:   t6    := d            scheduling
 9:   t7    := e
 6:   t5    := t3 + t4      no delays
10:   t8    := t6 + t7
 4:   x     := t3
 7:   y     := t5
11:   z     := t8
```


Lecture Compilation Methods SS 2013 / Slide 505

Objectives:

Restrictions for pipelining

In the lecture:

- Requirements of pipelining processors.
- Compiler reorders to meet the requirements, inserts nops (empty operations), if necessary.
- Some processors accept too close operations, delays the second one by a hardware interlock.
- Hardware bypasses may relax the requirements

Suggested reading:

Kastens / Übersetzerbau, Section 8.5.2

Questions:

- Why are no nops needed in this example?

Instruction Scheduling Algorithm for Pipelining

Algorithm: modified list scheduling:

Select from the ready list such that the selected operation

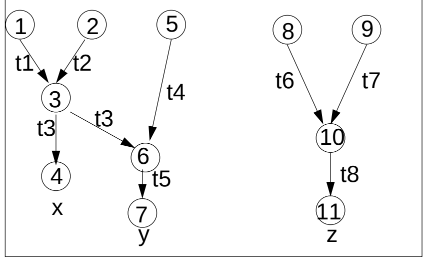
- has a sufficient **distance to all predecessors** in DDG
- has **many successors** (heuristic)
- has a **long path to the end** node (heuristic)

Insert an empty operation if none is selectable.

Ready list with additional information:

opr.	1	2	5	8	9	3	6	4	10	7	11
succ #	1	1	1	1	1	2	1	0	1	0	0
to end	3	3	2	2	2	2	1	1	1	0	0
sched. cycle	1	2	3	5	6	4	7	9	8	10	11

data dependence graph




```
cycle
 1     1:   t1    := a
 2     2:   t2    := b
 3     5:   t4    := c
 4     3:   t3    := t1 + t2      with
 5     8:   t6    := d            scheduling
 6     9:   t7    := e
 7     6:   t5    := t3 + t4
 8    10:   t8    := t6 + t7
 9     4:   x     := t3
10     7:   y     := t5
11    11:   z     := t8
```


Lecture Compilation Methods SS 2013 / Slide 506

Objectives:

Adapted list scheduling

In the lecture:

- Explain the algorithm using the example.
- Explain the selection criteria.

Suggested reading:

Kastens / Übersetzerbau, Section 8.5.2

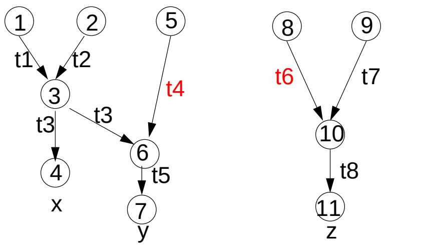
Reused registers: anti- and output-dependences

u ⟶ v **flow-dependence**:
u writes before v uses

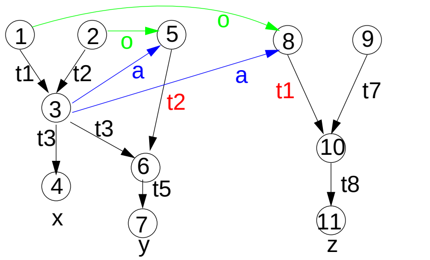
u $\xrightarrow{a}$ v **anti-dependence**:
u uses a value
before v overwrites it

u $\xrightarrow{o}$ v **output-dependence**:
u writes before v overwrites

DDG with symbolic registers ti
flow-dependences only



DDG with reused registers ti
flow, anti-, and output-dependences



Lecture Compilation Methods SS 2013 / Slide 506b

Objectives:

Understand anti- and output-dependences

In the lecture:

Explain anti- and output-dependences:

- Reuse of registers introduces new dependences

DDG with Loop Carried Dependences

Factorial computation:

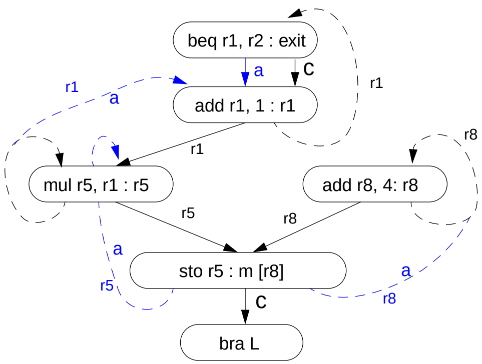
program:

```
i = 0; f = 1;
while ( i != n)
{   i = i + 1;
    f = f * i;

    m[i] = f;
}
```

seq. machine code:

```
L:  beq  r1, r2 : exit
    add  r1, 1  : r1
    mul  r5, r1 : r5
    add  r8, 4  : r8
    sto  r5 : m[r8]
    bra  L
```

Data dependence graph:



u ⟶ v **flow-dependence**:
u writes before v uses

u - - - → v **flow-dependence** into
subsequent iteration

u $\xrightarrow{a}$ v **anti-dependence**:
u uses a value
before v overwrites it

u $\xrightarrow{o}$ v **output-dependence**:
u writes before v overwrites

u $\xrightarrow{c}$ v **control-dependence**:
u has to be executed before v
(u or v may branch)

Lecture Compilation Methods SS 2013 / Slide 506d

Objectives:

Loop carried dependences

In the lecture:

Explain loop carried dependences

- the 4 kinds,
- they occur, because a new value is stored in the same register on every iteration,
- they are relevant, because we are going to merge operations of several iterations.

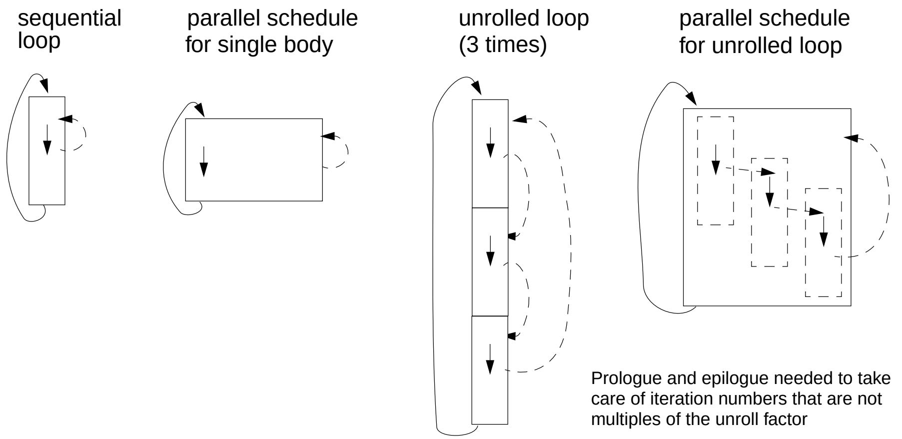
Questions:

- Explain why loops with arrays can have dependences into later iterations that are not the next one. Give an example.

Loop unrolling

Loop unrolling: A technique for parallelization of loops.

A single loop body does not exhibit enough parallelism => sparse schedule.
Schedule the code (copies) of several adjacent iterations together
=> more compact schedule



Prologue and epilogue needed to take care of iteration numbers that are not multiples of the unroll factor

Lecture Compilation Methods SS 2013 / Slide 506u

Objectives:

Understand the idea of loop unrolling

In the lecture:

- Compare the single body schedule to the schedule of the unrolled loop.
- Explain the consequences of loop carried dependences.

Suggested reading:

Kastens / Übersetzerbau, Section 8.5.2

Software Pipelining

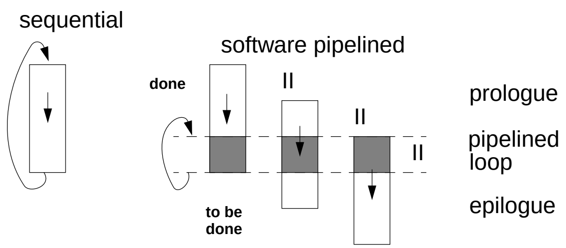
Software Pipelining: A technique for parallelization of loops.

A single loop body does not exhibit enough parallelism => sparse schedule.
Overlap the execution of several adjacent iterations => compact schedule

The **pipelined loop body**

has **each operation** of the original sequential body,
they belong to **several iterations**,
they are **tightly scheduled,**
its length is the **initiation interval II**,
is **shorter** than the original body.

Prologue, epilogue: initiation and finalization code



Lecture Compilation Methods SS 2013 / Slide 507

Objectives:

Understand the underlying idea

In the lecture:

- Explain the underlying idea
- II is both: length of the piplined loop and time between the start of two successive iterations.

Questions:

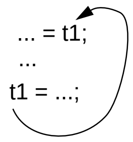
Explain:

- The shorter the initiation interval is, the greater is the parallelism, and the compacter is the schedule.

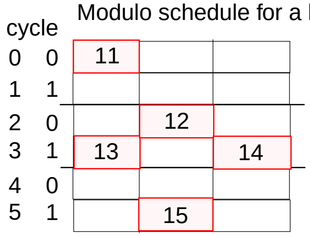
Transform Loops by Software Pipelining

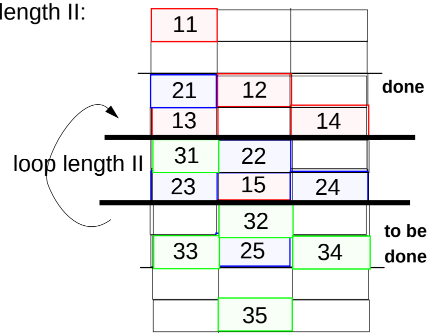
Technique:

1. **Data dependence graph** for the loop body, include **loop carried dependences**.
2. Chose a **small initiation interval II** - not smaller than #instructions / #FUs
3. Make a **„Modulo Schedule“** s for the loop body: Two instructions can not be scheduled on the same FU, i_1 in cycle c_1 and i_2 in cycle c_2, if c_1 mod II = c_2 mod II
4. If (3) does not succeed without conflict, increase II and repeat from 3
5. Allocate the instructions of s in the new loop of length II: i_j scheduled in cycle c_j is allocated to c_j mod II
6. Construct prologue and epilogue.



Modulo schedule for a loop body





Lecture Compilation Methods SS 2013 / Slide 508

Objectives:

Understand the technique

In the lecture:

- Explain the algorithm.
- Explain reasons for conflicts in step 4.

Questions:

Explain:

- The shorter the initiation interval is, the greater is the parallelism, and the compacter is the schedule.
- The transformed loop contains each instruction of the loop body exactly once.

Result of Software Pipelining

t	t_m		ADD	MUL	MEM	CTR
0	0	L:				beq r1, r2:exit
1	1		add r1, 1: r1			
2	0		add r8, 4 : r8	mul r5, r1 : r5		
3	1			... mul		
4	0				sto r5 : m r8	
5	1				... sto	
6	0					
7	1					bra L

4 dedicated FUs schedule of the loop body for II = 2

mul and sto need 2 cycles

add and sto in t_m=0, sto reads r8 before add writes it

bra not in cycle 6, it collides with beq: t_m=0

t	t_m		ADD	MUL	MEM	CTR
0	0					beq r1;r2:exit
1	1		add r1, 1 : r1			
2	0		add r8, 4 : r8	mul r5, r1 : r5		beq r1; r2 : ex
3	1		add r1, 1 : r1	... mul		
4	0	L:	add r8, 4 : r8	mul r5, r1 : r5	sto r5 : m r8	beq r1; r2 : ex
5	1		add r1, 1 : r1	... mul	... sto	bra L
6	1	ex:		... mul	... sto	
7	0				sto r5 : m r8	
8	1				... sto	
9	0					bra exit

prologue

software pipline with II = 2

epilogue

Lecture Compilation Methods SS 2013 / Slide 510

Objectives:

A software pipeline for a VLIW processor

In the lecture:

Explain

- the properties of the VLIW processor,
- the schedule,
- the software pipline,

Assignments:

- Make a table of run-times in cycles for n = 1, 2, ... iterations, and compare the figures without and with software pipelining.

5.2 / 6. Data Parallelism: Loop Parallelization

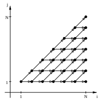
Regular loops on orthogonal data structures - parallelized for **data parallel** processors

Development steps (automated by compilers):

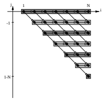
- **nested loops** operating on **arrays**, sequential execution of iteration space

```
DECLARE B[0..N,0..N+1]

FOR I := 1 ..N
    FOR J := 1 .. I
        B[I,J] :=
            B[I-1,J]+B[I-1,J-1]
    END FOR
END FOR
```

- analyze **data dependences** data-flow: definition and use of array elements

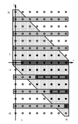


- **transform loops** keep data dependences forward in time



- **parallelize inner loop(s)** map to field or vector of processors

- **map arrays to processors** such that many accesses are local, transform index spaces



Lecture Compilation Methods SS 2013 / Slide 511

Objectives:

Overview

In the lecture:

Explain

- Application area: scientific computations
- goals: execute inner loops in parallel with efficient data access
- transformation steps

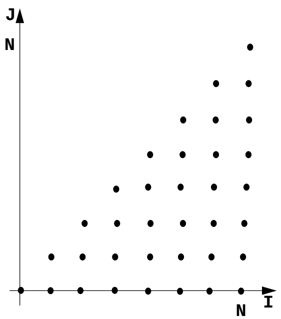
Iteration space of loop nests

Iteration space of a loop nest of depth n:

- **n-dimensional space of integral points** (polytope)
- each point (i_1, ..., i_n) represents an execution of the innermost loop body
- loop bounds are in general not known before run-time
- iteration need not have orthogonal borders
- iteration is elaborated sequentially

example:
computation of Pascal's triangle

```
DECLARE B[-1..N,-1..N]

FOR I := 0 .. N
    FOR J := 0 .. I
        B[I,J] :=
            B[I-1,J]+B[I-1,J-1]
    END FOR
END FOR
```



Lecture Compilation Methods SS 2013 / Slide 512

Objectives:

Understand the notion of iteration space

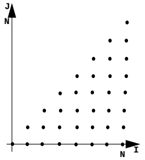
In the lecture:

- Explain the iteration space of the example.
- Show the order of elaboration of the iteration space.
- If the step size is greater than 1 the iteration space has gaps - the polytope is not convex.

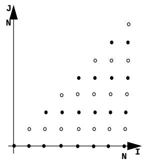
Questions:

- Draw an iteration space that has step size 3 in one dimension.

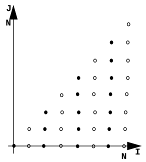
Examples for Iteration spaces of loop nests



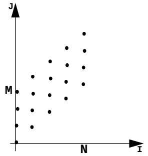

```
FOR I := 0 .. N
   FOR J := 0 .. I
```



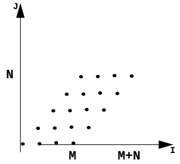

```
FOR I := 0 .. N
   FOR J := 0..I BY 2
```




```
FOR I := 0..N BY 2
   FOR J := 0 .. I
```




```
FOR I := 0 .. N
   FOR J := I..I+M

M = 3, N = 4
```




```
FOR I := 0 .. M+N
   FOR J := max(0, I-M)..
            min (I, N)
```


Lecture Compilation Methods SS 2013 / Slide 512a

Objectives:

Relate loop nests to iteration spaces

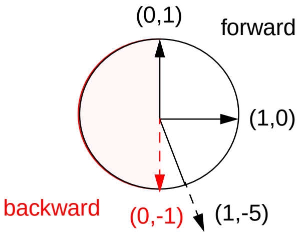
In the lecture:

- Explain the iteration spaces of the examples

Data Dependences in Iteration Spaces

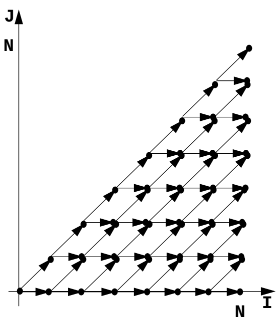
Data dependence from iteration point i1 to i2:

- Iteration **i**1 computes a value that is used in iteration **i2** (flow dependence)
- relative **dependence vector**
  d = **i2 - i1** = ($i2_1$ - $i1_1$, ..., $i2_n$ - $i1_n$)
  holds for all iteration points except at the border
- Flow-dependences can **not be directed against the execution order**, can not point backward in time: each dependence vector must be **lexicographically positive**, i. e. **d** = (0, ..., 0, d_i, ...), $d_i > 0$



Example:
Computation of Pascal´s triangle

```
DECLARE B[-1..N,-1..N]

FOR I := 0 .. N
   FOR J := 0 .. I
      B[I,J] :=
         B[I-1,J]+B[I-1,J-1]
   END FOR
END FOR
```



Lecture Compilation Methods SS 2013 / Slide 513

Objectives:

Understand dependences in loops

In the lecture:

Explain:

- Vector representation of dependences,
- examples,
- admissable directions graphically

Questions:

- Show different dependence vectors and array accesses in a loop body which cause dependences of given vectors.

Loop Transformation

The **iteration space** of a loop nest is transformed to **new coordinates**. Goals:

- **execute innermost loop(s) in parallel**
- improve **locality** of data accesses;
  in space: use storage of executing processor,
  in time: reuse values stored in cache
- **systolic** computation and communication scheme

Data dependences must **point forward in time**, i.e.
lexicographically positive and
not within parallel dimensions

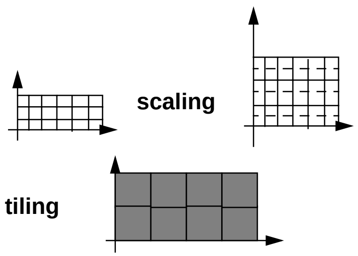
linear basic transformations:

- **Skewing**: add iteration count of an outer loop to that of an inner one
- **Reversal**: flip execution order for one dimension
- **Permutation**: exchange two loops of the loop nest

SRP transformations (next slides)

non-linear transformations, e. g.

- **Scaling**: **stretch the iteration space** in one dimension, causes gaps
- **Tiling**: introduce **additional inner loops** that **cover tiles** of fixed size



Lecture Compilation Methods SS 2013 / Slide 514

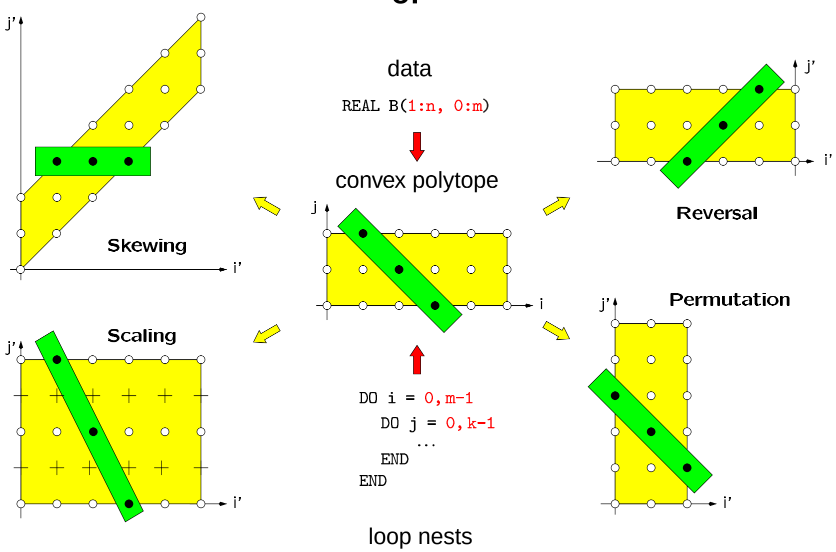
Objectives:

Overview

In the lecture:

- Explain the goals.
- Show admissable directions of dependences.
- Show diagrams for the transformations.

Transformations of



Lecture Compilation Methods SS 2013 / Slide 514a

Objectives:

Visualize the transformations

In the lecture:

- Give concrete loops for the diagrams.
- Show how the dependence vectors are transformed.
- Skewing and scaling do not change the order of execution; hence, they are always applicable.

Questions:

- Give dependence vectors for each transformation, which are still valid after the transformation.

Transformations defined by matrices

Transformation matrices: systematic transformation, check dependence vectors

Reversal $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} i \\ -j \end{pmatrix} = \begin{pmatrix} i' \\ j' \end{pmatrix}$

Skewing $\begin{pmatrix} 1 & 0 \\ f & 1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} i \\ f*i+j \end{pmatrix} = \begin{pmatrix} i' \\ j' \end{pmatrix}$

Permutation $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} j \\ i \end{pmatrix} = \begin{pmatrix} i' \\ j' \end{pmatrix}$

Lecture Compilation Methods SS 2013 / Slide 514b

Objectives:

Understand the matrix representation

In the lecture:

- Explain the principle.
- Map concrete iteration points.
- Map dependence vectors.
- Show combinations of transformations.

Questions:

- Give more examples for skewing transformations.

Reversal

Iteration count of one loop is negated, that dimension is enumerated backward

general transformation matrix

$$\begin{pmatrix} 1 & & & & & 0 \\ & \ldots & & & & \\ & & 1 & & & \\ & & & \mathbf{-1} & & \\ & & & & 1 & \\ 0 & & & & & \ldots \; 1 \end{pmatrix}$$

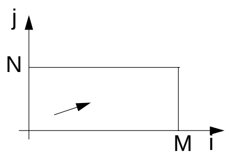
2-dimensional:

loop variables old new

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} i \\ -j \end{pmatrix} = \begin{pmatrix} ir \\ jr \end{pmatrix}$$

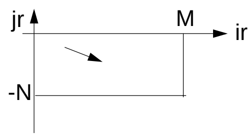
```
for i = 0 to M
   for j = 0 to N
      ...
```

```
for ir = 0 to M
   for jr = -N to 0
      ...
```



original

transformed



Lecture Compilation Methods SS 2013 / Slide 515

Objectives:

Understand reversal transformation

In the lecture:

- Explain the effect of reversal transformation.
- Explain the notation of the transformation matrix.
- There may be no dependences in the direction of the reversed loop - they would point backward after the transformation.

Questions:

- Show an example where reversal enables loop fusion.

Skewing

The **iteration count** of an outer loop is **added to the count of an inner loop;**
iteration space is shifted; **execution order** of iteration points **remains unchanged**

general transformation matrix:

$$\begin{pmatrix} 1 & & & & & \\ & \ldots & & & & 0 \\ & & 1 & & & \\ & & \mathbf{f} & 1 & & \\ 0 & & & & 1 \ldots & \\ & & & & & 1 \end{pmatrix}$$

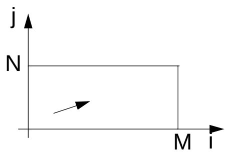
2-dimensional:

loop variables
old new

$$\begin{pmatrix} 1 & 0 \\ f & 1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} i \\ f*i+j \end{pmatrix} = \begin{pmatrix} is \\ js \end{pmatrix}$$

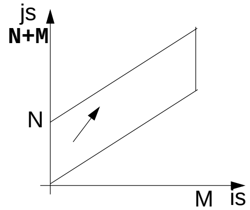
```
for i = 0 to M
   for j = 0 to N
      ...
```

original




```
for is = 0 to M
   for js = f*is to N+f*is
      ...
```

transformed



Lecture Compilation Methods SS 2013 / Slide 516

Objectives:

Understand skewing transformation

In the lecture:

- Explain the effect of a skewing transformation.
- Skewing is always applicable.
- Skewing can enable loop permutation

Questions:

- Show an example where skewing enables loop permutation.

Permutation

Two loops of the loop nest are interchanged; the iteration space is flipped;
the **execution order** of iteration points **changes;** new dependence vectors must be legal.

general transformation matrix:

$$\begin{array}{cc} & \\ i & \\ & \\ j & \\ & \\ & \end{array}\begin{pmatrix} 1 & & & & & \\ & \mathbf{0} & \mathbf{1} & & & 0 \\ & & 1 & & & \\ & \mathbf{1} & \mathbf{0} & & & \\ 0 & & & 1 \ldots & & \\ & & & & & 1 \end{pmatrix}$$
$$\begin{array}{cc} i & j \end{array}$$

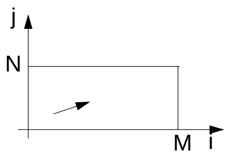
2-dimensional:

loop variables
old new

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} j \\ i \end{pmatrix} = \begin{pmatrix} ip \\ jp \end{pmatrix}$$

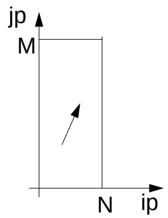
```
for i = 0 to M
   for j = 0 to N
      ...
```

original




```
for ip = 0 to N
   for jp = 0 to M
      ...
```

transformed



Lecture Compilation Methods SS 2013 / Slide 517

Objectives:

Understand loop permutation

In the lecture:

- Explain the effect of loop permutation.
- Show effect on dependence vectors.
- Permutation often yields a parallelizable innermost loop.

Questions:

- Show an example where permutation yields a parallelizable innermost loop.

Use of Transformation Matrices

- Transformation matrix **T** defines **new iteration counts** in terms of the old ones: **T * i = i´**

e. g. Reversal
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} i \\ -j \end{pmatrix} = \begin{pmatrix} i' \\ j' \end{pmatrix}$$

- Transformation matrix **T** transforms old **dependence vectors** into new ones: **T * d = d´**

e. g.
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} * \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

- inverse Transformation matrix $\mathbf{T}^{-1}$ defines **old iteration counts** in terms of new ones, for transformation of index expressions in the loop body: $\mathbf{T}^{-1} * \mathbf{i}' = \mathbf{i}$

e. g.
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} * \begin{pmatrix} i' \\ j' \end{pmatrix} = \begin{pmatrix} i' \\ -j' \end{pmatrix} = \begin{pmatrix} i \\ j \end{pmatrix}$$

- **concatenation of transformations** first $\mathbf{T_1}$ then $\mathbf{T_2}$: $\mathbf{T_2} * \mathbf{T_1} = \mathbf{T}$

e. g.
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Lecture Compilation Methods SS 2013 / Slide 518

Objectives:

Learn to Use the matrices

In the lecture:

- Explain the 4 uses with examples.
- Transform a loop completely.

Questions:

- Why do the dependence vectors change under a transformation, although the dependence between array elements remains unchanged?

Inequalities Describe Loop Bounds

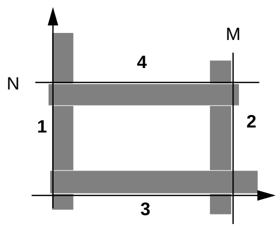
The bounds of a loop nest are described by a **set of linear inequalities**.
Each **inequality separates the space** in „inside and outside of the iteration space“:

$$\mathbf{B * i \le c}$$

$$\begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} \le \begin{pmatrix} 0 \\ M \\ 0 \\ N \end{pmatrix}$$

example 1

1 $-i \le 0$
2 $i \le M$
3 $-j \le 0$
4 $j \le N$

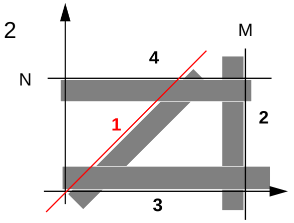


$$\begin{pmatrix} -1 & 1 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} \le \begin{pmatrix} 0 \\ M \\ 0 \\ N \end{pmatrix}$$

example 2

1 $-i + j \le 0$
2 $i \le M$
3 $-j \le 0$
4 $j \le N$

transformed



1, 4: $j \le \min(i, N)$ **1+ 3:** $0 \le i$
3: $0 \le j$ **2:** $i \le M$

positive factors represent **upper** bounds
negative factors represent **lower** bounds

Lecture Compilation Methods SS 2013 / Slide 519

Objectives:

Understand representation of bounds

In the lecture:

- Explain matrix notation.
- Explain graphic interpretation.
- There can be arbitrary many inequalities.

Questions:

- Give the representations of other iteration spaces.

Transformation of Loop Bounds

The inverse of a transformation matrix $\mathbf{T}^{-1}$ transforms a set of inequalities: $\mathbf{B} * \mathbf{T}^{-1}\, \mathbf{i'} \le \mathbf{c}$

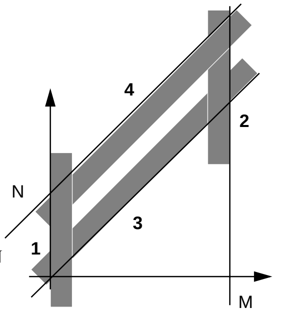
skewing $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ inverse $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$

$$\underbrace{\begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix}}_{B} * \underbrace{\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}}_{T^{-1}} = \underbrace{\begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ -1 & 1 \end{pmatrix}}_{B * T^{-1}}$$

example 1
new bounds:

$$\underbrace{\begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ -1 & 1 \end{pmatrix}}_{B * T^{-1}} * \underbrace{\begin{pmatrix} i' \\ j' \end{pmatrix}}_{i'} \le \underbrace{\begin{pmatrix} 0 \\ M \\ 0 \\ N \end{pmatrix}}_{c}$$

1 $-i' \le 0$
2 $i' \le M$
3 $i' - j' \le 0$
4 $-i' + j' \le N$



Lecture Compilation Methods SS 2013 / Slide 520

Objectives:

Understand the transformation of bounds

In the lecture:

- Explain how the inequalities are transformed

Questions:

- Compute further transformations of bounds.

Example for Transformation and Parallelization of a Loop

```
for i = 0 to N
   for j = 0 to M
      a[i, j] = (a[i, j-1] + a[i-1, j]) / 2;
```

Parallelize the above loop.

1. Draw the iteration space.
2. Compute the dependence vectors and draw examples of them into the iteration space.
   Why can the inner loop not be executed in parallel?
3. Apply a skewing transformation and draw the iteration space.
4. Apply a permutation transformation and draw the iteration space.
   Explain why the inner loop now can be executed in parallel.
5. Compute the matrix of the composed transformation and
   use it to transform the dependence vectors.
6. Compute the inverse of the transformation matrix and
   use it to transform the index expressions.
7. Specify the loop bounds by inequalities and
   transform them by the inverse of the transformation matrix.
8. Write the complete loops with new loop variables ip and jp and new loop bounds.

Lecture Compilation Methods SS 2013 / Slide 521

Objectives:

Exercise the method for an example

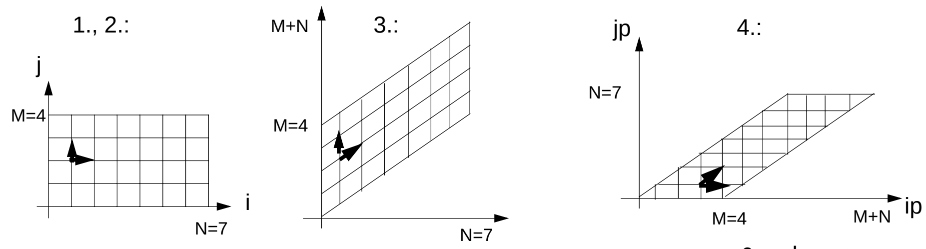
In the lecture:

- Explain the steps of the transformation.
- Solution on C-5.22

Questions:

- Are there other transformations that lead to a parallel inner loop?

Solution of the Transformation and Parallelization Example



5.: $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

6.: Inverse $\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$

7. Bounds:

orig.: $\begin{array}{cc} B & c \\ \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 0 \\ N \\ 0 \\ M \end{pmatrix} \end{array}$

new: $\begin{array}{c} B * T^{-1} \\ \begin{pmatrix} 0 & -1 \\ 0 & 1 \\ -1 & 1 \\ 1 & -1 \end{pmatrix} \end{array}$

1 $-jp \le 0$ **1, 3 => $0 \le ip$**
2 $jp \le N$ **2, 4 => $ip \le M+N$**
3 $-ip+jp \le 0$ **1, 4 => $\max(0, ip-M) \le jp$**
4 $ip - jp \le M$ **2, 3 => $jp \le \min(ip, N)$**

8.
```
for ip = 0 to M+N
   for jp = max (0, ip-M) to min (ip, N)
      a[jp, ip-jp] = (a[jp, ip-jp-1] + a[jp-1, ip-jp]) / 2;
```


Lecture Compilation Methods SS 2013 / Slide 522

Objectives:

Solution for C-60

In the lecture:

Explain

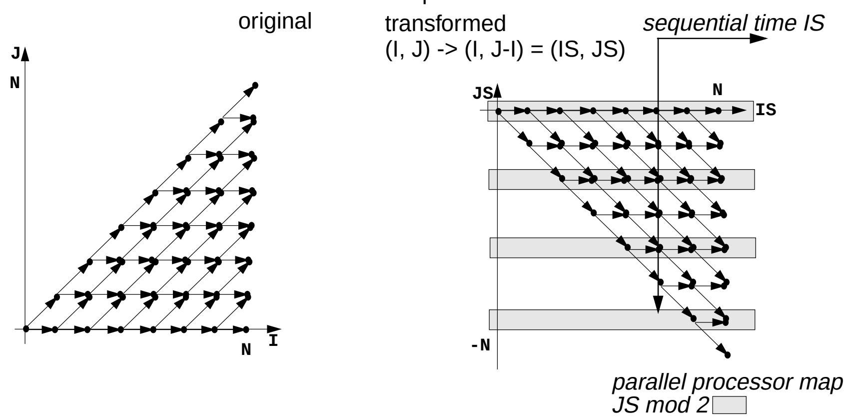
- the bounds of the iteration spaces,
- the dependence vectors,
- the transformation matrix and its inverse,
- the conditions for being parallelizable,
- the transformation of the index expressions
- the transformation of the loop bounds.

Questions:

- Describe the transformation steps.

Transformation and Parallelization

Iteration space




```
DECLARE B[-1..N,-1..N]

FOR I := 0 .. N
   FOR J := 0 .. I
      B[I,J] :=
         B[I-1,J]+B[I-1,J-1]
   END FOR
END FOR
```

```
DECLARE B[-1..N,-1..N]

FOR IS := 0.. N
   FOR JS := -IS .. 0
      B[IS,JS+IS] :=
         B[IS-1,JS+IS]+B[IS-1,JS-1+IS]
   END FOR
END FOR
```

Lecture Compilation Methods SS 2013 / Slide 523

Objectives:

Example for parallelization

In the lecture:

- Explain skewing transformation: f = -1
- Inner loop in parallel.
- Explain the time and processor mapping.
- mod 2 folds the arbitrary large loop dimension on a fixed number of 2 processors.

Questions:

- Give the matrix of this transformation.
- Use it to compute the dependence vectors, the index expressions, and the loop bounds.

Data Mapping

Goal:
Distribute array elements over processors, such that as many **accesses as possible are local.**

Index space of an array:
n-dimensional space of integral index points (polytope)

- **same properties as iteration space**
- same mathematical model
- same **transformations** are applicable (Skewing, Reversal, Permutation, ...)
- **no restrictions** by data dependences

Lecture Compilation Methods SS 2013 / Slide 524

Objectives:
Reuse model of iteration spaces

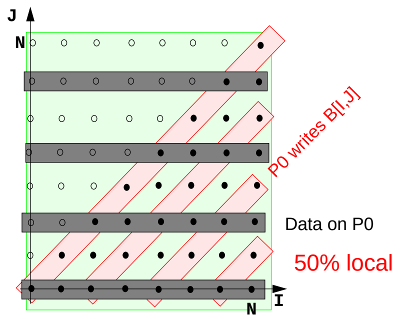
In the lecture:
Explain, using examples of index spaces

Questions:
- Draw an index space for each of the 3 transformations.

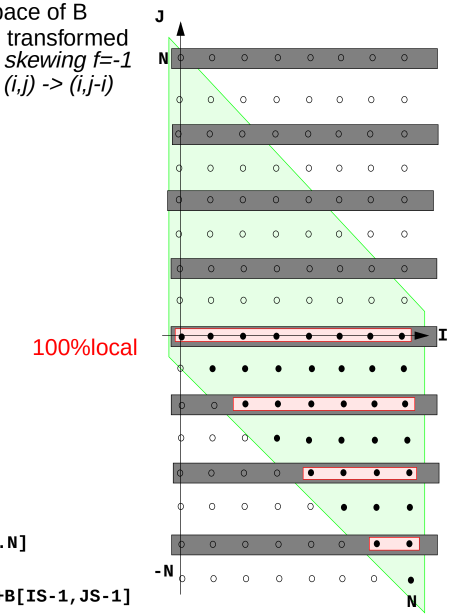
Data distribution for parallel loops

index space of B

original



transformed
skewing f=-1
(i,j) -> (i,j-i)




```
DECLARE B[-1..N,-1..N]

FOR IS := 0.. N
    FOR JS := -IS .. 0
        B[IS,JS+IS] :=
            B[IS-1,JS+IS]+B[IS-1,JS-1+IS]
    END FOR
END FOR
```

```
DECLARE B[-1..N,-N..N]
...
    B[IS,JS] :=
        B[IS-1,JS-1]+B[IS-1,JS-1]
```

Lecture Compilation Methods SS 2013 / Slide 525

Objectives:
The gain of an index transformation

In the lecture:
Explain
- local and non-local accesses,
- the index transformation,
- the gain of locality,
- unused memory because of skewing.

Questions:
- How do you compute the index transformation using a transformation matrix?

Check Your Knowledge (1)

C-6.1

Optimization, CFA:

1. Explain graphs that are used in program analysis.
2. Which optimizing transformations need analysis of execution pathes?
3. Which optimizing transformations do not need analysis of execution pathes?
4. Give an example for a pair of transformations such that one enables the other.
5. Define the control-flow graph. Describe transformations on the CFG.
6. Define the dominator relation. What is it used for?
7. Describe an algorithm for computing dominator sets.
8. Define natural loops.
9. What is the role of the loop header and of the pre-header.
10. Show a graph that has a cycle but no natural loop.
11. Define induction variables, and explain the transformation technique.

Lecture Compilation Methods SS 2011 / Slide 601

Objectives:

Support repetition and understanding of the material

In the lecture:

- Answer some questions:
- Let some questions be answered.

Check Your Knowledge (2)

C-6.2

Optimization, DFA:

12. Describe the schema for DFA equations for the four problem categories.
13. Explain the relation of the meet operator, the paths in the graph, and the DFA solutions.
14. Describe the DFA problem reaching definitions.
15. Describe the DFA problem live variables.
16. Describe the DFA problem available expressions.
17. Describe the DFA problem copy propagation.
18. Describe the DFA problem constant propagation.
19. Describe the iterative DFA algorithm; its termination; its complexity.
20. Describe an heuristic improvement of the iterative DFA algorithm.
21. Extend constant propagation to interval propagation for bounds checks. Explain the interval lattice.
22. What is the role of lattices in DFA?
23. Describe lattices that are common for DFA.

Lecture Compilation Methods SS 2011 / Slide 602

Objectives:

Support repetition and understanding of the material

In the lecture:

- Answer some questions:
- Let some questions be answered.

Check Your Knowledge (3)

C-6.3

Object Oriented Program Analysis:

24. Describe techniques to reduce the number of arcs in call graphs.
25. Describe call graphs for object oriented programs.
26. Describe techniques to reduce the number of arcs in object oriented call graphs.

Code Generation, Storage mapping:

27. Explain the notions of storage classes, relative addresses, alignment, overlay.
28. Compare storage mapping of arrays by pointer trees to mapping on contiguous storage.
29. Explain storage mapping of arrays for C. What is different for C, for Fortran?
30. For what purpose are array descriptors needed? What do they contain?
31. What is the closure of a function? In which situation is it needed?
32. Why must a functional parameter in Pascal be represented by a pair of pointers?
33. What does an activation record contain?
34. Explain static links in the run-time stack. What is the not-most-recent property?
35. How do C, Pascal, and Modula-2 ensure that the run-time stack discipline is obeyed?
36. Why do threads need a separate run-time stack each?

Lecture Compilation Methods SS 2011 / Slide 603

Objectives:

Support repetition and understanding of the material

In the lecture:

- Answer some questions:
- Let some questions be answered.

Check Your Knowledge (4)

C-6.4

37. Explain the code for function calls in relation to the structure of activation records.
38. Explain addressing relative to activation records.
39. Explain sequences for loops.
40. Explain the translation of short circuit evaluation of boolean expressions. Which attributes are used?
41. Explain code selection by covering trees with translation patterns.
42. Explain a technique for tree pattern selection using 3 passes.
43. Explain code selection using parsing. What is the role of the grammar?

Register Allocation

44. How is register windowing used for implementation of function calls?
45. Which allocation technique is applied for which program context?
46. Explain register allocation for expression trees. Which attributes are used?
47. How is spill code minimized for expression trees?
48. Explain register allocation for basic blocks? Relate the spill criteria to paging techniques.
49. Explain register allocation by graph coloring. What does the interference graph represent?
50. Explain why DFA life-time analysis is needed for register allocation by graph coloring.

Lecture Compilation Methods SS 2011 / Slide 604

Objectives:

Support repetition and understanding of the material

In the lecture:

- Answer some questions:
- Let some questions be answered.

Check Your Knowledge (5)

Instruction Scheduling

51. What does instruction scheduling mean for VLIW, pipeline, and vector processors?
52. Explain the kinds of arcs of DDGs (flow, anti, output).
53. What are loop carried dependences?
54. Explain list scheduling for parallel FUs. How is the register need modelled? Compare it to Belady’s register allocation technique.
55. How is list scheduling applied for arranging instructions for pipeline processors?
56. Explain the basic idea of software pipelining. What does the initiation interval mean?

Loop Parallelization

57. Explain dependence vectors in an iteration space. What are the admissible directions for sequential and for parallelized innermost loops?
58. What is tiling, what is scaling?
59. Explain SRP transformations.
60. How are the transformation matrices used?
61. How are loop bounds transformed?
62. Parallelize the inner loop of a nest that has dependence vectors (1,0) and (0, 1)?

Objectives:

Support repetition and understanding of the material

In the lecture:

- Answer some questions:
- Let some questions be answered.