Data-Flow Analysis

Data-flow analysis (DFA) provides information about
how the **execution of a program may manipulate its data**.

Many different problems can be formulated as **data-flow problems**, for example:

- Which assignments to variable **v** may influence a use of **v** at a certain program position?
- Is a variable **v** used on any path from a program position **p** to the exit node?
- The values of which expressions are available at program position **p**?

Data-flow problems are stated in terms of

- **paths through the control-flow graph** and
- **properties of basic blocks**.

Data-flow analysis provides information for **global optimization.**

Data-flow analysis does not know

- which input values are provided at run-time,
- which branches are taken at run-time.

Its results are to be interpreted **pessimistic**

Data-Flow Equations

A data-flow problem is stated as a **system of equations** for a control-flow graph.

System of Equations for **forward problems** (propagate information along control-flow edges):

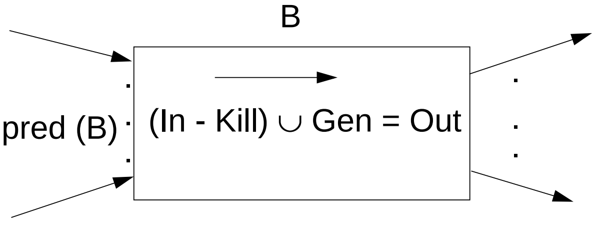
Example **Reaching definitions:**
A definiton **d** of a variable **v** reaches the begin of a block **B** if **there is a path** from **d** to **B** on which **v** is not assigned again.

In, Out, Gen, Kill represent analysis information:
sets of statements,
sets of variables,
sets of expressions
depending on the analysis problem

2 equations for each basic block:

$$\text{Out}(B) = f_B(\text{In}(B)) = \text{Gen}(B) \cup (\text{In}(B) - \text{Kill}(B))$$

$$\text{In}(B) = \underset{h \in \text{pred}(B)}{\Theta} \text{Out}(h)$$



In, Out	**variables** of the system of equations for each block
Gen, Kill	a pair of **constant sets** that characterize a block w.r.t. the DFA problem

Θ meet operator; e. g. Θ **=** ∪ for „reaching definitions", Θ **=** ∩ for „available expressions"

Specification of a DFA Problem

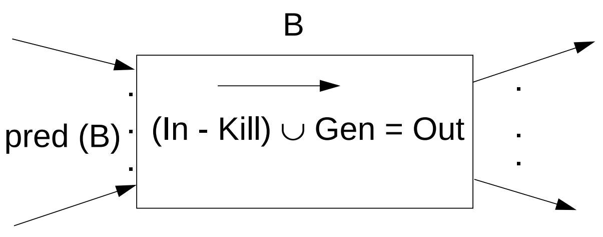
Specification of reaching definitions:

1. **Description**:
   A definiton **d** of a variable **v** reaches the begin of a block **B**
   if **there is a path** from **d** to **B** on which **v** is not assigned again.
2. It is a **forward problem**.
3. The **meet operator** is union.
4. The **analysis information** in the sets are assignments at certain program positions.
5. **Gen (B)**:
   contains all definitions **`d: v = e;`** in **B**,
   such that **v** is not defined after **d** in **B**.
6. **Kill (B)**:
   if **v** is assigned in **B**, then **Kill(B)**
   contains all definitions **`d: v = e;`**
   of blocks different from **B**.

2 equations for each basic block:

$$\text{Out (B)} = f_B \text{ (In (B))}$$
$$= \text{Gen (B)} \cup \text{(In (B) - Kill (B))}$$

$$\text{In (B)} = \underset{h \in \text{pred(B)}}{\Theta} \text{Out (h)}$$



Variants of DFA Problems

- **forward** problem:
  DFA information flows **along the control flow**
  In(B) is determined by Out(h) of the predecessor blocks

  backward problem (see C-2.23):
  DFA information flows **against the control flow**
  Out(B) is determined by In(h) of the successor blocks

- **union** problem:
  problem description: „there is a path";
  meet operator is $\Theta = \cup$
  solution: minimal sets that solve the equations

  intersect problem:
  problem description: „for all paths"
  meet operator is $\Theta = \cap$
  solution: maximal sets that solve the equations

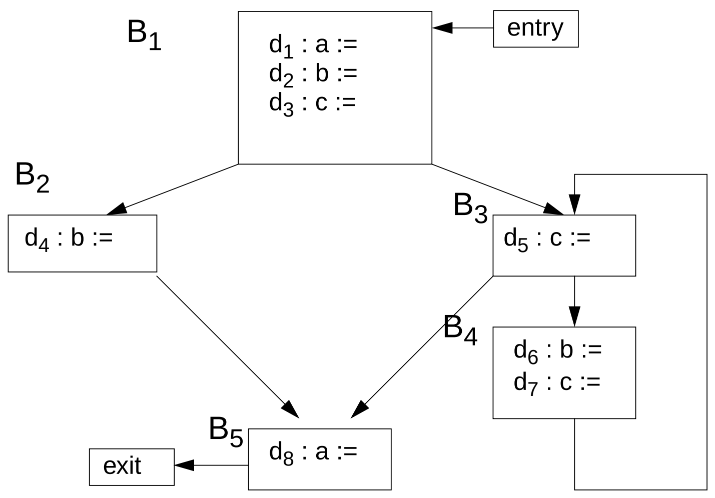
- **optimization information**: **sets of** certain statements, of variables, of expressions.

Further classes of DFA problems over general lattices instead of sets are not considered here.

Example Reaching Definitions

Gen (B):
contains all definitions **`d: v = e;`** in **B**, such that **v** is not defined after **d** in **B**.

Kill (B):
contains all definitions **`d: v = e;`** in blocks different from **B**, such that B has a definition of **v**.



Description of DFA-Problem

	Gen	**Kill**
B_1	d_1, d_2, d_3	d_4, d_5, d_6, d_7, d_8
B_2	d_4	d_2, d_6
B_3	d_5	d_3, d_7
B_4	d_6, d_7	d_2, d_3, d_4, d_5
B_5	d_8	d_1

DFA-Solution

In	**Out**
$\emptyset$	d_1, d_2, d_3
d_1, d_2, d_3	d_1, d_3, d_4
d_1, d_2, d_3, d_6, d_7	d_1, d_2, d_5, d_6
d_1, d_2, d_5, d_6	d_1, d_6, d_7
$d_1, d_2, d_3, d_4, d_5, d_6$	$d_2, d_3, d_4, d_5, d_6, d_8$

Iterative Solution of Data-Flow Equations

Input: the CFG; the sets Gen(B) and Kill(B) for each basic block B
Output: the sets In(B) and Out(B)

Algorithm:
```
repeat
   stable := true;
   for all B ≠ entry   {*}
   do begin
      for all V ∈ pred(B) do
         In(B):= In(B) Θ Out(V);
      oldout:= Out(B);
      Out(B):= Gen(B) ∪ (In(B)-Kill(B));
      stable:= stable and Out(B)=oldout
   end
until stable
```

Initialization

Union: empty sets
```
for all B do
begin
   In(B):=∅;
   Out(B):=Gen(B)
end;
```

Intersect: full sets
```
for all B do
begin
   In(B) := U;
   Out(B):=
      Gen(B)∪
          (U - Kill(B))
end;
```

Complexity: $O(n^3)$ with n number of basic blocks
$O(n^2)$ if $|$**pred (B)**$|$ $\leq$ **k << n** for all B

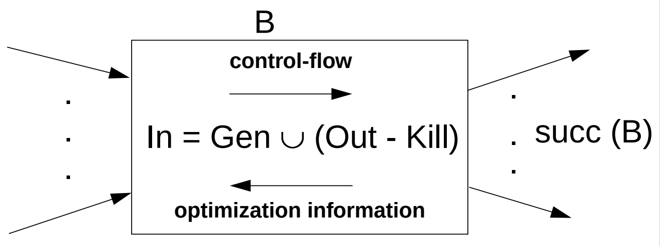
Backward Problems

System of Equations for **backward problems**
propagate information against control-flow edges:

2 equations for each basic block:

$$\mathrm{In}(B) = f_B(\mathrm{Out}(B)) = \mathrm{Gen}(B) \cup (\mathrm{Out}(B) - \mathrm{Kill}(B))$$

$$\mathrm{Out}(B) = \underset{h \in \mathrm{succ}(B)}{\Theta} \mathrm{In}(h)$$



Example **Live variables:**

1. Description: Is variable **v** alive at a given point **p** in the program, i. e. **is there a path** from **p** to the exit where **v** is used but not defined before the use?
2. backward problem
3. optimization information: sets of variables
4. meet operator: Θ **=** $\cup$ union
5. Gen (B): variables that are used in B, but not defined before they are used there.
6. Kill (B): variables that are defined in B, but not used before they are defined there.

Important Data-Flow Problems

1. **Reaching definitions:** A definiton **d** of a variable **v** reaches the beginning of a block **B** if there is a path from **d** to **B** on which **v** is not assigned again.
   DFA variant: forward; union; set of assignments
   Transformations: use-def-chains, constant propagation, loop invariant computations
2. **Live variables:** Is variable **v** alive at a given point **p** in the program, i. e. there is a path from **p** to the exit where **v** is used but not defined before the use.
   DFA variant: backward; union; set of variables
   Transformations: eliminate redundant assignments
3. **Available expressions:** Is expression **e** computed on every path from the entry to a program position **p** and none of its variables is defined after the last computation before **p**.
   DFA variant: forward; intersect; set of expressions
   Transformations: eliminate redundant computations
4. **Copy propagation:** Is a copy assignment **c: x = y** redundant, i.e. on every path from **c** to a use of **x** there is no assignment to **y**?
   DFA variant: forward; intersect; set of copy assignments
   Transformations: remove copy assignments and rename use
5. **Constant propagation:** Has variable **x** at position **p** a known value, i.e. on every path from the entry to **p** the last definition of **x** is an assignment of the same known value.
   DFA variant: forward; combine function; vector of values
   Transformations: substitution of variable uses by constants

Algebraic Foundation of DFA

DFA performs computations on a **lattice (dt. Verband)** of values, e. g. bit-vectors representing finite sets. It guarantees termination of computation and well-defined solutions. see [Muchnick, pp 223-228]

A **lattice L** is a set of values with two operations: $\cap$ **meet** and $\cup$ **join**

Required properties:

1. **closure**: $x, y \in L$ implies $x \cap y \in L$, $x \cup y \in L$
2. **commutativity**: $x \cap y = y \cap x$ and $x \cup y = y \cup x$
3. **associativity**: $(x \cap y) \cap z = x \cap (y \cap z)$ and $(x \cup y) \cup z = x \cup (y \cup z)$
4. **absorption**: $x \cap (x \cup y) = x = x \cup (x \cap y)$
5. unique elements **bottom** $\perp$, **top** T:
   $x \cap \perp = \perp$ and $x \cup T = T$

In most DFA problems only a **semilattice** is used with L, $\cap$, $\perp$ or L, $\cup$, T

Partial order defined by meet, $x \subseteq y$: $x \cap y = x$ (transitive, antisymmetric, reflexive); defined by join: $x \supseteq y$: $x \cup y = x$

Some DFA Lattices

2

Bool

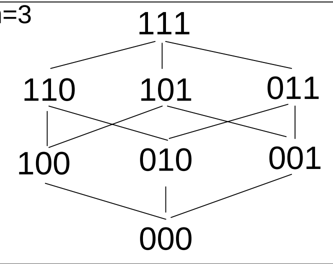
$\cap$ = and
$\cup$ = or

T = true
|
$\perp$ = false

1

Bit-Vector BV$^{n=3}$

$\cap$ = bitwise and
$\cup$ = bitwise or



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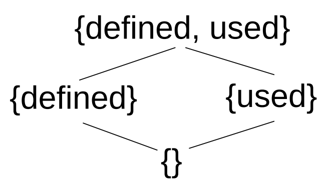
Variable usage



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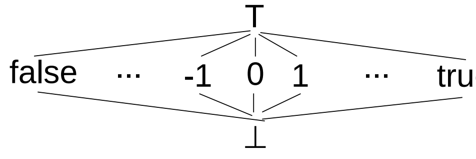
ICP Integer Constant Propagation Lattice



$n \cap \perp = \perp$ $\quad n \cap n = n$ $\quad n \cap m = \perp$ if $n \neq m$
$n \cup T = T$ $\quad n \cup n = n$ $\quad n \cup m = T$ if $n \neq m$

5

Range Lattice: $[lo, hi] \in (Z \cup \{-\infty, \infty\})^2$

$\perp$ = [ ] empty range, T = $[-\infty, \infty]$,
$x \subseteq y$: x is contained in y

```
∩: [l1, h1] ∩ [l2, h2] = x
     let l = max (l1, l2),
     h = min (h1, h2),
     x = if h < l then ⊥ else [l, h]
```

$\cup$: $[l1, h1] \cup [l2, h2] =$
$[\min(l1, l2), \max(h1, h2)]$

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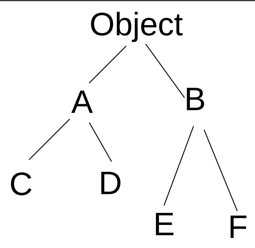
Semilattice of types

$\cup$: $x \cup y$ = smallest common supertype of x and y



Monotone Functions Over Lattices

The **effects of program constructs on DFA information** are described by functions over a suitable lattice,

e. g. the function for basic block B_3 on C-2.22:

$f_3(<x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8>) = <x_1\ x_2\ 0\ x_4\ 1\ x_6\ 0\ x_8> \in BV^8$

Gen-Kill pair encoded as function

f: L $\rightarrow$ L is a **monotone function** over the lattice L if
$\forall$ x, y $\in$ L: x $\subseteq$ y $\Rightarrow$ f(x) $\subseteq$ f(y)

Finite height of the lattice and **monotonicity** of the functions guarantee **termination** of the algorithms.

Fixed points z of the function f, with f(z) = z, is a solution of the set of DFA equations.

MOP: Meet over all paths solution is desired, i. e. the „best" with respect to L

MFP: Maximum fixed point is computed by algorithms, if functions are monotone

If the functions f are additionally **distributive**, then **MFP = MOP**.
f: L $\rightarrow$ L is a **distributive function** over the lattice L if
$\forall$ x, y $\in$ L: f(x $\cap$ y) = f(x) $\cap$ f(y)

Variants of DFA Algorithms

Heuristic improvement:

Goal: propagate changes in the In and Out sets as fast as possible.
Technique: visit CFG nodes in topological order in the outer for-loop {*}.
Then the number of iterations of the outer repeat-loop is only determined by back edges in the CFG

Algorithm for backward problems:

Exchange In and Out sets symmetrically in the algorithm of C-2.22b.
The nodes should be visited in topological order as if the directions of edges were flipped.

Hierarchical algorithms, interval analysis:

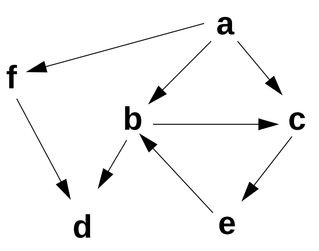
Regions of the CFG are considered nodes of a CFG on a higher level.
That abstraction is recursively applied until a single root node is reached.
The Gen, Kill sets are combined in upward direction;
the In, Out sets are refined downward.

Program Analysis: Call Graph (context-insensitive)

Nodes: defined functions

Arc g -> h: function g contains a call h(),
i. e. a call g() **may** cause the execution of a call h()

```
void a ()  {...b()...c()...f()...}
void b ()  {...d()...c()...}
void c()   {...e()...}
void d()   {...}
void e()   {...v++;...b()...}
void f()   {...d()...}
```


$$\begin{tikzcd} & & & a \arrow[llld] \arrow[ld] \arrow[d] \\ f \arrow[d] & & b \arrow[r] \arrow[ld] & c \arrow[d] \\ & d & & e \arrow[lu] \end{tikzcd}$$

Analysis of structure:
b, c, e are recursive;
a, d, f are non-recursive

Propagation of properties:
assume a call e() may **modify a global variable** v
then calls a(), b(), c() may indirectly cause modification of v

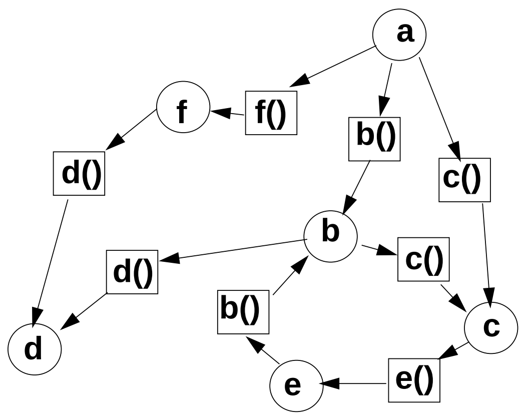
```
v = f(); cnt = 0; while(...){...b(); cnt += v;}
```

Program Analysis: Call Graph (context-sensitive)

Nodes: defined functions and calls (bipartite)

Arc g -> h: function g contains a call h(),i.e a call g() **may** cause the execution of a call h()
or call g() leads to function g

```
void a ()  {...b()...c()...f()...}
void b ()  {...d()...c()...}
void c()   {...e()...}
void d()   {...}
void e()   {...v++;...b()...}
void f()   {...d()...}
```



Calls of the same function in different contexts are distinguished by **different nodes**, e.g. the call of c in a and in b.

Analysis can be **more precise** in that aspect.

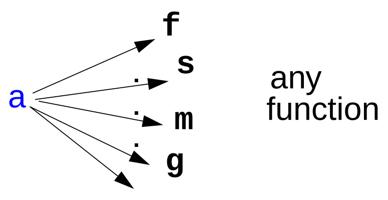
Calls Using Function Variables

Contents of **function variables is assigned at run-time**.

Static analysis does not know (precisely) which function is called.

Call graph has to assume that **any function may be called**.

```
void a()
  {...(*h)(0.3, 27)...}
```



Analysis for a better approximation of potential callees:

only those functions which

1. fit to the type of h
2. are assigned somewhere in the program
3. can be derived from the reaching definitions at the call

```
void m (int j) {...}

void g (float x, int i) {...}

...k = m;... f(g); ...

void a()
  {  void (*h)(float,int) = g;
     ...
     if(...) h = s;

     ...(*h)(0.3, 27)...
  }
```

Analysis of Object-Oriented Programs

Aspects specific for object-oriented analysis:

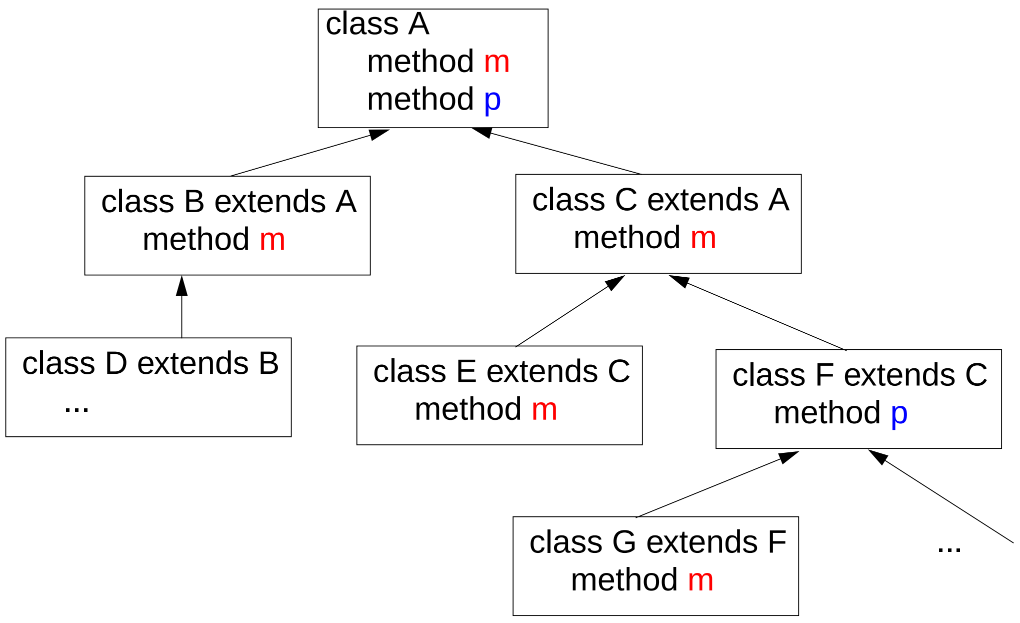
1. **hierarchy of classes and interfaces**
   specifies a complex **system of subtypes**
2. **hierarchy of classes and interfaces**
   specifies **inheritance and overriding** relation for methods
3. **dynamic method binding**
   for method calls `v.m(...)` the **callee is determined at run-time**
   good object-oriented style relies on that feature
4. **many small methods** are typical object-oriented style
5. **library use and reuse of modules**
   complete program contains many **unused classes and methods**

Static predictions for dynamically bound method calls
are essential for most analyses

Class Hierarchy Graph

Node: **class or interface**

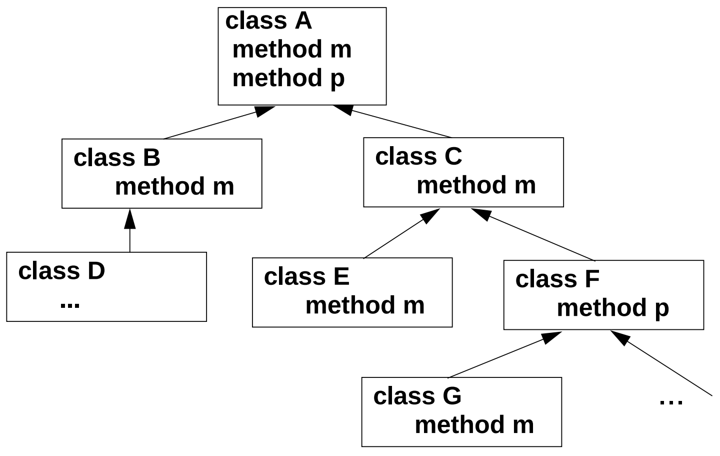
Arc a -> b: a is **subclass of** b or a **implements interface** b



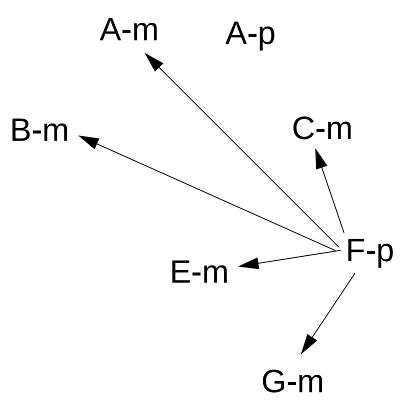
Object-Oriented Call Graph

Node: **implemented method**,
identified by class name, method name: X-a

Arc X-a -> Y-b: method **X-a contains a call** v.b(...) that
may be bound to Y-b



Call graph for F-p containing v.m(...)



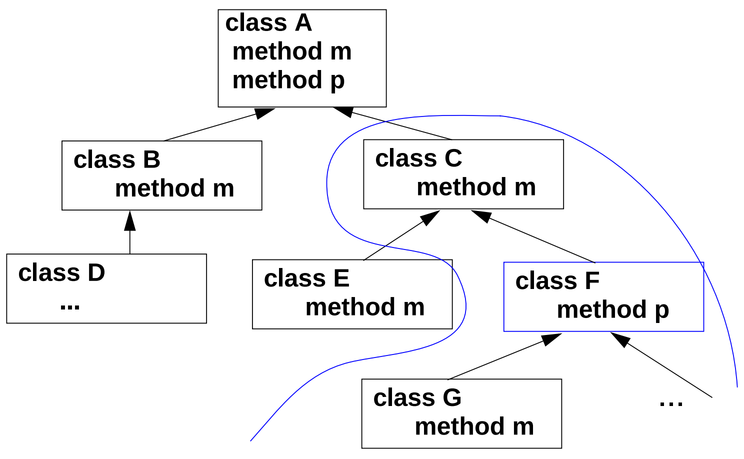
Call graph: **any method m** may be bound to that call in F-p
(compare to function variables)
analysis yields better approximations

Call Graphs Constructed by Class Hierarchy Analysis (CHA)

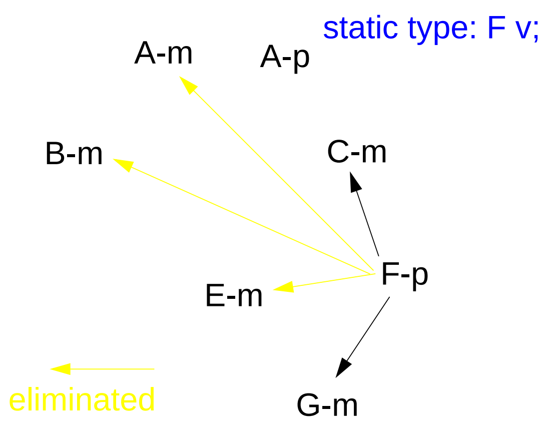
The call graph is reduced to a set of **reachable methods** using the **class hierarchy** and the **static type of the receiver** expression in the call:

If a method F-p **is reachable** and
if it contains a **dynamically bound call v.m(...)** and
T is the static type of v,

then every method **m that is inherited by T or by a subtype of T is also reachable**, and arcs go from F-p to them.



Call graph for F-p containing v.m(...)



Refined Approximations for Call Graph Construction

Class Hierarchy Analysis (CHA): (see C-2.32)

Rapid Type Analysis (RTA):

As CHA, but only methods of those classes C are considered which are instantiated (**`new C()`**) in a reachable method.

Reaching Type Analysis:

Approximations of run-time types is propagated through a graph: nodes represent variables, arcs represent copy assignments.

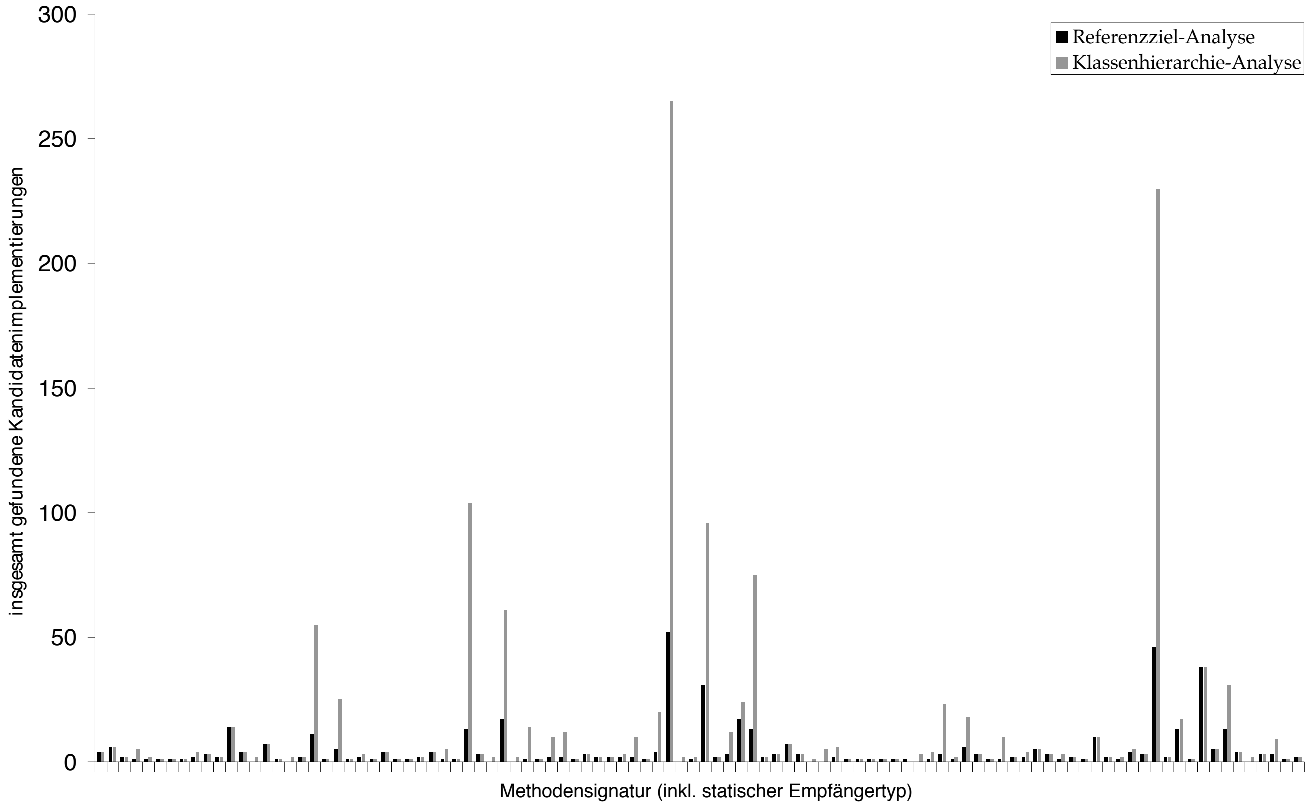
Declared Type Analysis:
one node T represents all variables declared to have type T

Variable Type Analysis:
one node V represents a single variable

Points-to Analysis:

Information on object identities is propagated through the control-flow graph

Results of Analysis of Dynamically Bound Calls



Modules of a Toolset for Program Analysis

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analysis module	purpose	category
ClassMemberVisibility	examines visibility levels of declarations	visualization
MethodSizeStatistics	examines length of method implementations in bytecode operations and frequency of different bytecode operations	
ExternalEntities	histogram of references to program entities that reside outside a group of classes	
InheritanceBoundary	histogram of lowest superclass outside a group of classes	
SimpleSetterGetter	recognizes simple access methods with bytecode patterns	
MethodInspector	decomposes the raw bytecode array of a method implementation into a list of instruction objects	auxiliary analysis
ControlFlow	builds a control flow graph for method implementations	fundamental analyses
Dominator	constructs the dominator tree for a control flow graph	
Loop	uses the dominator tree to augment the control flow graph with loop and loop nesting information	
InstrDefUse	models operand accesses for each bytecode instruction	
LocalDefUse	builds intraprocedural def / use chains	
LifeSpan	analyzes lifeness of local variables and stack locations	
DefUseTypeInfo	infers type information for operand accesses	analysis of incomplete programs
Hierarchy	class hierarchy analysis based on a horizontal slice of the hierarchy	
PreciseCallGraph	builds call graph based on inferred type information, copes with incomplete class hierarchy	
ParamEscape	transitively traces propagation of actual parameters in a method call (escape = leaves analyzed library)	
ReadWriteFields	transitive liveness and access analysis for instance fields accessed by a method call	

Table 0-1. Analysis plug-ins in our framework

[Michael Thies: Combining Static Analysis of Java Libraries with Dynamic Optimization, Dissertation, Shaker Verlag, April 2001]