

# Compilation Methods

Prof. Dr. Uwe Kastens

Summer 2013

## 1 Introduction

### Objectives

The students are going to learn

- what the main tasks of the **synthesis part of optimizing compilers** are,
- how **data structures and algorithms** solve these tasks systematically,
- what can be achieved by **program analysis and optimizing transformations**,

### Prerequisites

- Constructs and properties of programming languages
- What does a compiler know about a program?
- How is that information represented?
- Algorithms and data structures of the analysis parts of compilers (frontends)

Main aspects of the lecture **Programming Languages and Compilers** (PLaC, BSc program)  
<http://ag-kastens.upb.de/lehre/material/plac>

## Syllabus

Week	Chapter	Topic
1	1 Introduction	Compiler structure
	2 Optimization	Overview: Data structures, program transformations
2		Control-flow analysis
3		Loop optimization
4, 5		Data-flow analysis
6		Object oriented program analysis
7	3 Code generation	Storage mapping
		Run-time stack, calling sequence
8		Translation of control structures
9		Code selection by tree pattern matching
10, 11	4 Register allocation	Expression trees (Sethi/Ullman)
		Basic blocks (Belady)
		Control flow graphs (graph coloring)
12	5 Code Parallelization	Data dependence graph
13		Instruction Scheduling
14		Loop parallelization
15	Summary	

## References

Course material:

**Compilation Methods:** <http://ag-kastens.upb.de/lehre/material/compil>

**Programming Languages and Compilers:** <http://ag-kastens.upb.de/lehre/material/plac>

Books:

U. Kastens: **Übersetzerbau**, Handbuch der Informatik 3.3, Oldenbourg, 1990; (sold out)

K. Cooper, L. Torczon: **Engineering A Compiler**, Morgan Kaufmann, 2003

S. S. Muchnick: **Advanced Compiler Design & Implementation**, Morgan Kaufmann Publishers, 1997

A. W. Appel: **Modern Compiler Implementation in C**, 2nd Edition  
 Cambridge University Press, 1997, (in Java and in ML, too)

W. M. Waite, L. R. Carter: **An Introduction to Compiler Construction**, Harper Collins, New York, 1993

M. Wolfe: **High Performance Compilers for Parallel Computing**, Addison-Wesley, 1996

A. V. Aho, M. S. Lam, R. Sethi, J. D. Ullman: **Compilers - Principles, Techniques, & Tools**, 2nd Ed, Pearson International Edition (Paperback), and Addison-Wesley, 2007

# Course Material in the Web: HomePage

C-1.6

Lecture Compilation Methods SS 2013

Slides	Assignments
<ul style="list-style-type: none"> <li>Chapters</li> <li>Slides</li> <li>Printing</li> </ul>	<ul style="list-style-type: none"> <li>Assignments</li> <li>Printing</li> </ul>
Organization	Resources
<ul style="list-style-type: none"> <li>General Information</li> <li>News</li> </ul>	<ul style="list-style-type: none"> <li>Objectives</li> <li>Literature</li> <li>Internet Links</li> <li>Material: Programming Languages and Compilers</li> </ul>

Veranstaltungs-Nummer: L079.05810  
Generiert mit Camelot | Probleme mit Camelot? | Geändert am: 19.02.2013

© 2013 bei Prof. Dr. Uwe Kastens

# Course Material in the Web: Organization

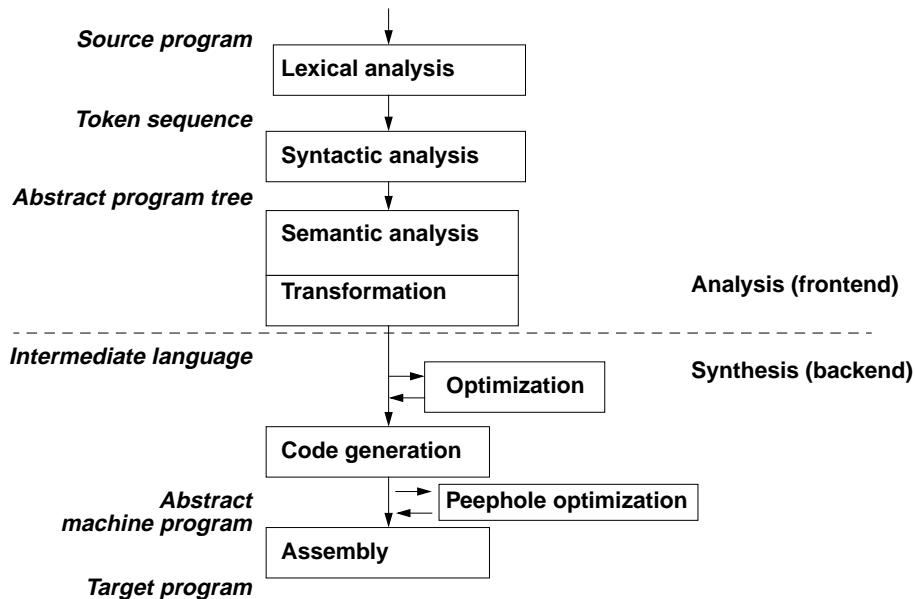
C-1.6a

Lecturer
<p><b>Prof. Dr. Uwe Kastens:</b></p> <p><b>Office hours</b></p> <ul style="list-style-type: none"> <li>Wed 16.00 - 17.00 F2.308</li> <li>Thu 11.00 - 12.00 F2.308</li> </ul>
Hours
<p><b>Lecture</b></p> <ul style="list-style-type: none"> <li>V2 Fr 11:15 - 12:45 F1.110</li> </ul> <p><b>Start date:</b> Fr Apr 12, 2013</p> <p><b>Tutorials</b></p> <ul style="list-style-type: none"> <li>Ü2 Fr 13:15 - 14:45, F1.110, even weeks</li> </ul> <p><b>Dates:</b> 19.04., 03.05., 17.05., 31.05., 14.06., 28.06., 12.07.</p>
Examination
<p>This course is examined in an oral examination, which in general is held in English. It may be held in German, if the candidate does not need the certificate of an English examination.</p> <p>In the study program Master of Computer Science the examination for this course is part of a module examination which covers two courses. It may contribute to the module examination of one of the modules III.1.2 (type A), III.1.5 (type A), or III.1.6 (type B). Please follow the instructions for examination registration or in German zur Prüfungsanmeldung</p> <p>In other study programs a single oral examination for this course may be taken.</p> <p>In any case a candidate has to register for the examination in PAUL and has to ask for a date for the exam via eMail to me.</p> <p>The next time spans I offer for oral exams are July 31 to Aug 01, 2013, and Oct 09 to 11, 2013.</p>
Homework
<p><b>Homework assignments</b></p> <ul style="list-style-type: none"> <li>Homework assignments are published every other week on Fridays.</li> </ul>

© 2013 bei Prof. Dr. Uwe Kastens

# Compiler Structure and Interfaces

C-1.7



© 2002 bei Prof. Dr. Uwe Kastens

# 2 Optimization

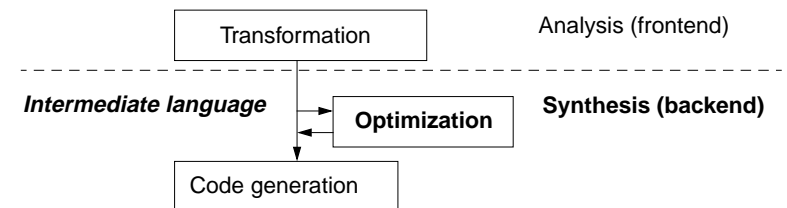
C-2.1

**Objective:**  
Reduce run-time and / or code size of the program,  
**without changing its observable effects.**  
Eliminate redundant computations, simplify computations.

**Input:** Program in intermediate language

**Task:** find redundancies (**analysis**)  
improve the code (**optimizing transformations**)

**Output:** Improved program in intermediate language



© 2013 bei Prof. Dr. Uwe Kastens

## Overview on Optimizing Transformations

C-2.2

### Name of transformation:

### Example for its application:

- Algebraic simplification** of expressions  
2\*3.14 => 6.28    x+0 => x    x\*2 => shift left    x\*\*2 => x\*x
- Constant propagation** (dt. Konstantenweitergabe)  
constant values of variables propagated to uses:    `x = 2; ... y = x * 5;`
- Common subexpressions** (gemeinsame Teilausdrücke)  
avoid re-evaluation, if values are unchanged    `x = a*(b+c); ... y = (b+c)/2;`
- Dead variables** (überflüssige Zuweisungen)  
eliminate redundant assignments    `x = a + b; ... x = 5;`
- Copy propagation** (überflüssige Kopieranweisungen)  
substitute use of x by y    `x = y; ... ; z = x;`
- Dead code** (nicht erreichbarer Code)  
eliminate code, that is never executed    `b = true; ... if (b) x = 5; else y = 7;`

© 2016 bei Prof. Dr. Uwe Kastens

## Overview on Optimizing Transformations (continued)

C-2.2a

### Name of transformation:

### Example for its application:

- Code motion** (Code-Verschiebung)  
move computations to cheaper places    `if (c) x = (a+b)*2; else x = (a+b)/2;`
- Function inlining** (Einsetzen von Aufrufen)  
substitute call of small function by a computation over the arguments    `int Sqr (int i) { return i * i; }  
x = Sqr (b*3)`
- Loop invariant code**  
move invariant code before the loop    `while (b) {... x = 5; ...}`
- Induction variables in loops**  
transform multiplication into incrementation    `i = 1; while (b) { k = i*3; f(k); i = i+1; }`

© 2019 bei Prof. Dr. Uwe Kastens

## Program Analysis for Optimization

C-2.3

### Static analysis:

**static properties** of program structure and of **every execution**;  
**safe, pessimistic assumptions**  
where input and dynamic execution paths are not known

### Context of analysis - the larger the more information:

Expression	local optimization
Basic block	local optimization
procedure (control flow graph)	global intra-procedural optimization
program module (call graph)	global inter-procedural optimization
separate compilation	
complete program	optimization at link-time or at run-time

### Analysis and Transformation:

Analysis provides preconditions for **applicability of transformations**

Transformation may change analysed properties,  
may **inhibit or enable** other transformations

**Order** of analyses and transformations **is relevant**

© 2011 bei Prof. Dr. Uwe Kastens

## Program Analysis in General

C-2.4

**Program text** is systematically analyzed to exhibit  
**structures** of the program,  
**properties** of program entities,  
**relations** between program entities.

### Objectives:

#### Compiler:

- Code improvement
- automatic parallelization
- automatic allocation of threads

#### Software engineering tools:

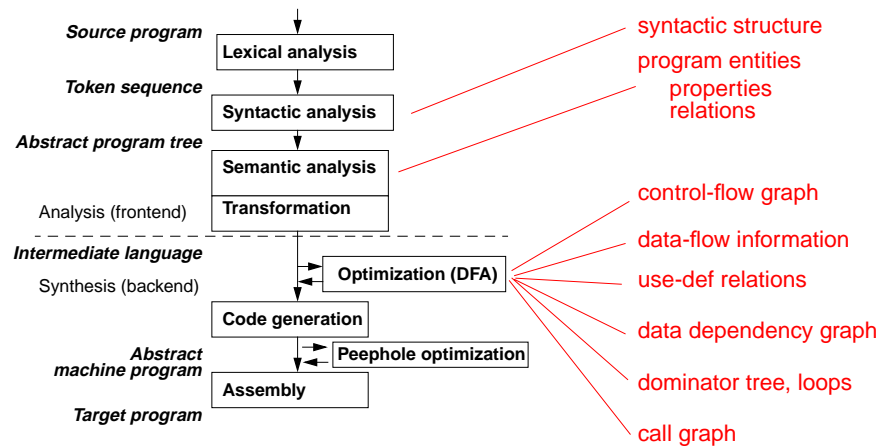
- program understanding
- software maintenance
- evaluation of software qualities
- reengineering, refactoring

**Methods** for program analysis stem from **compiler construction**

© 2004 bei Prof. Dr. Uwe Kastens

# Overview on Program Analysis in Compilers

C-2.5



© 2016 bei Prof. Dr. Uwe Kastens

# Basic Blocks

C-2.6

**Basic Block (dt. Grundblock):**  
Maximal sequence of instructions that can be entered only at the first of them and exited only from the last of them.

## Begin of a basic block:

- procedure entry
- target of a branch
- instruction after a branch or return (must have a label)

## Function calls

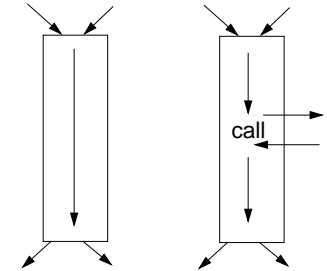
are usually not considered as a branch, but as operations that have effects

## Local optimization

considers the context of one single basic block (or part of it) at a time.

## Global optimization:

Basic blocks are the nodes of control-flow graphs.



© 2013 bei Prof. Dr. Uwe Kastens

# Example for Basic Blocks

C-2.7

A C function that computes Fibonacci numbers:

```
int fib (int m)
{ int f0 = 0, f1 = 1, f2, i;
  if (m <= 1)
    return m;
  else
  { for(i=2; i<=m; i++)
    { f2 = f0 + f1;
      f0 = f1;
      f1 = f2;
    }
    return f2;
  } }
```

**if-condition** belongs to the preceding basic block

**while-condition** does not belong to the preceding basic block

Intermediate code with basic blocks:

[Muchnick, p. 170]

1	receive m	
2	f0 <- 0	
3	f1 <- 1	<b>B1</b>
4	if m <= 1 goto L3	
5	i <- 2	<b>B3</b>
6	L1: if i <= m goto L2	<b>B4</b>
7	return f2	<b>B5</b>
8	L2: f2 <- f0 + f1	
9	f0 <- f1	
10	f1 <- f2	<b>B6</b>
11	i <- i + 1	
12	goto L1	
13	L3: return m	<b>B2</b>

© 2013 bei Prof. Dr. Uwe Kastens

# Control-Flow Graph (CFG)

C-2.8

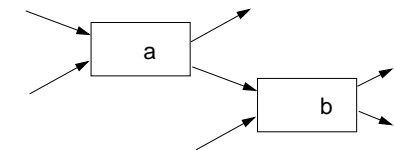
A **control-flow graph, CFG** (dt. Ablaufgraph) represents the control structure of a function

**Nodes:** **basic blocks** and 2 unique nodes **entry** and **exit**.

**Edge a -> b:** control may flow from the end of a to the begin of b

## Fundamental data structure for

- control flow analysis
- structural transformations
- code motion
- data-flow analysis (DFA)



© 2002 bei Prof. Dr. Uwe Kastens

## Example for a Control-flow Graph

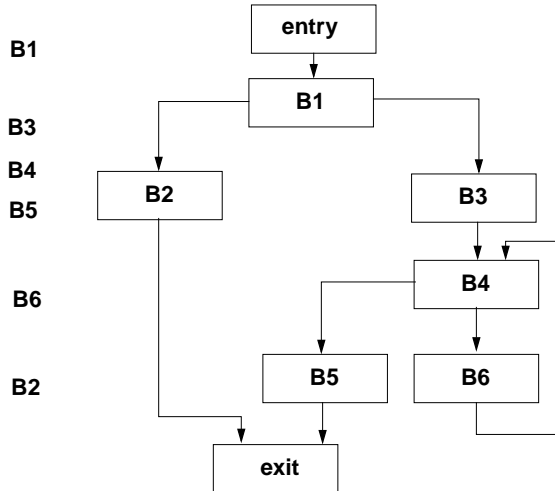
C-2.9

Intermediate code with basic blocks:

```

1  receive m
2  f0 <- 0
3  f1 <- 1
4  if m <= 1 goto L3
5  i <- 2
6  L1: if i <= m goto L2
7  return f2
8  L2: f2 <- f0 + f1
9  f0 <- f1
10 f1 <- f2
11 i <- i + 1
12 goto L1
13 L3: return m
    
```

Control-flow graph:  
[Muchnick, p. 172]



© 2012 bei Prof. Dr. Uwe Kastens

## Control-Flow Analysis

C-2.10

Compute **properties on the control-flow** based on the CFG:

- **dominator relations:**  
properties of paths through the CFG
- **loop recognition:**  
recognize loops - independent of the source language construct
- **hierarchical reduction of the CFG:**  
a region with a unique entry node on the one level is a node of the next level graph

Apply **transformations** based on control-flow information:

- **dead code elimination:**  
eliminate unreachable subgraphs of the CFG
- **code motion:**  
move instructions to better suitable places
- **loop optimization:**  
loop invariant code, strength reduction, induction variables

© 2011 bei Prof. Dr. Uwe Kastens

## Dominator Relation on CFG

C-2.11

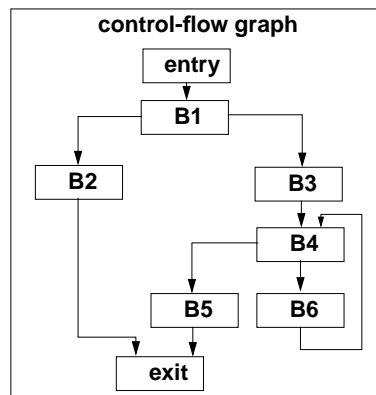
Relation over nodes of a CFG, characterizes paths through CFG,  
used for loop recognition, code motion

**a dominates b (a dom b):**

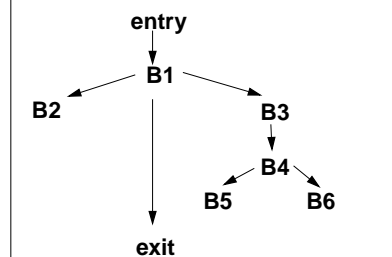
a is on every path from the entry node to b (reflexive, transitive, antisymmetric)

**a is immediate dominator of b (a idom b):**

a dom b and a ≠ b, and there is no c such that c ≠ a, c ≠ b, a dom c, c dom b.



**Tree of (immediate) dominators**  
(dom is transitive closure of the tree)



© 2011 bei Prof. Dr. Uwe Kastens

## Immediate Dominator Relation is a Tree

C-2.11a

Every node has a unique immediate dominator.

The dominators of a node are linearly ordered by the idom relation.

Proof by contradiction:

Assume:

a ≠ b, a dom n, b dom n and  
not (a dom b) and not (b dom a)

Then there are paths in the CFG

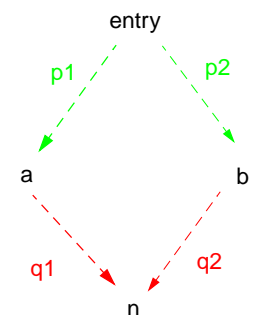
- p1: from entry to a not touching b, since not (b dom a)
- p2: from entry to b not touching a, since not (a dom b)
- q1: from a to n not touching b, since a dom n and not (a dom b)
- q2: from b to n not touching a, since b dom n and not (b dom a)

Hence, there is a path p1-q1 from entry via a to n not touching b.

That is a contradiction to the assumption b dom n.

Hence, n has a unique immediate dominator, either a or b.

CFG



© 2013 bei Prof. Dr. Uwe Kastens

### Dominator Computation

Algorithm computes the sets of dominators  
 Domin(n) for all nodes n ∈ N of a CFG:

```

for each n ∈ N do Domin(n) = N;
Domin(entry) = {entry};

repeat
  for each n ∈ N - {entry} do
    T = N;
    for each p ∈ pred(n) do
      T = T ∩ Domin(p);
    Domin(n) = {n} ∪ T;
  until Domin is unchanged
    
```

Symmetric relation for backward analysis:

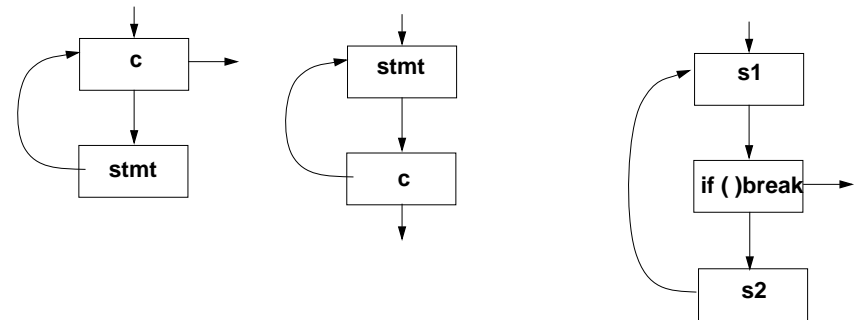
**a postdominates b (a pdom b):**  
 a is on every path from b to the exit node (reflexive, transitive, antisymmetric)

### Loop Recognition: Structured Loops

while (c) stmt;

do stmt; while (c);

do s1; if ( )break; s2; while (true);



### Loop Recognition: Natural Loops

**Back edge t->h** in a CFG: head h dominates tail t (h dom t).

**Natural loop of a back edge t->h:**

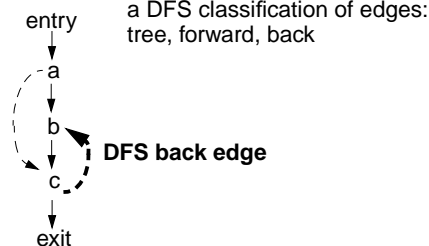
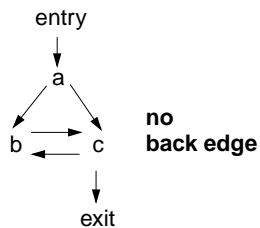
set S of nodes such that S contains h, t and all nodes from which t can be reached without passing through h. h is the **loop header**.

**Iterative computation** of the natural loop for t->h:

add predecessors of nodes in S according to the formula:

$$S = \{h, t\} \cup \{p \mid \exists a (a \in S \setminus \{h\} \wedge p \in \text{pred}(a))\}$$

This definition of **back edges** is stronger than that of **DFS back edges**:



### Example for Loop Recognition

back edge:

- 4 -> 3
- 6 -> 2
- 7 -> 2
- 6 -> 6

natural loop:

- S<sub>1</sub> = {3,4}
- S<sub>2</sub> = {2, 3, 4, 5, 6}
- S<sub>3</sub> = {2, 3, 4, 5, 7}
- S<sub>4</sub> = {6}

loops are

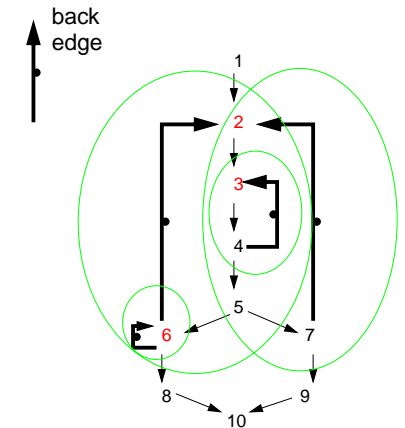
- disjoint
- nested
- non-nested,

$$S_1 \cap S_4 = \emptyset$$

$$S_1 \subset S_2$$

$$S_2, S_3$$

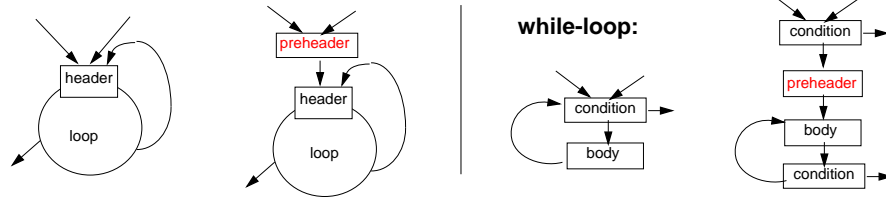
but have the same loop header, are comprised into one loop



## Loop Optimization

C-2.15

- Introduce a **preheader** for a loop, as a place for loop invariant computations: a new, empty basic block that lies on every path to the loop header, but is not iterated:



- move **loop invariant computations** to the preheader: check use-def-chains: if an expression E contains no variables that are defined in the loop, then replace E by a temporary variable t, and compute t = E; in the preheader.
- eliminate **redundant bounds-checks**: propagate value intervals using the same technique as for constant propagation (see DFA) Example in Pascal:

```
var a: array [1..10] of integer;
    i: integer;
```

```
for i := 1 to 10 do a[i] := i;
```

- induction variables, strength reduction**: see next slide

© 2013 bei Prof. Dr. Uwe Kastens

## Loop Induction Variables

C-2.16

Induction variables may occur in any loop - not only in `for` loops.

**Induction variable i:**

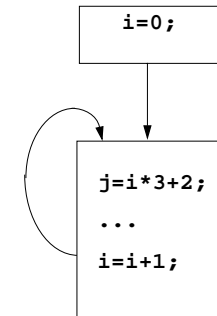
i is incremented (decremented) by a constant value c on every iteration.

**Basic induction variable i:**

There is exactly one definition  $i = i + c$ ; or  $i = i - c$ ; that is executed on every path through the loop.

**Dependent induction variable j:**

j depends on induction variable i by a linear function  $i * a + b$  represented by (i, a, b).



© 2002 bei Prof. Dr. Uwe Kastens

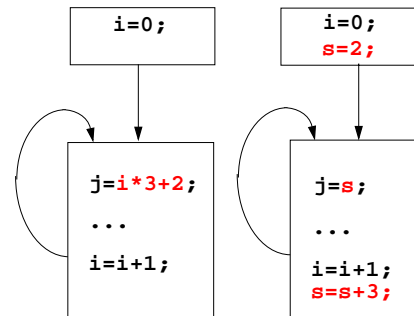
## Transformation of Induction Variables

C-2.17

**Transformation** of dependent induction variables:

- For each (i, a, b) create a temporary variable s.
- Initialize  $s = i * a + b$ ; in the preheader.
- Replace  $i * a + b$  in the loop by s.
- Add  $s = s + c*a$ ; behind the increment of i

j: (i, 3, 2)



**Strength reduction:**

Replace a costly operation (multiplication) by a cheaper one (addition).

**Linear increment of array address computation** (next slide)

© 2006 bei Prof. Dr. Uwe Kastens

## Examples for Transformations of Induction Variable

C-2.17a

```
do
  k = i*3+1;
  f (5*k);
  /* x = a[i]; compiled: */
  x = cont(start+i*elsize);
  i = i + 2;
while (Ek)
```

basic induction variable:

i: c = 2

dependent induction variables:

k: (i, 3, 1)

arg: (k, 5, 0)

ind: (i, elsize, start)

```
sk = i*3+1;
sarg = sk*5;
sind = start + i*elsize;
do
  k = sk;
  f (sarg);
  x = cont (sind);
  i = i + 2;
  sk = sk + 6;
  sarg = sarg + 30;
  sind = sind + 2*elsize;
while (Ek)
```

© 2006 bei Prof. Dr. Uwe Kastens

## Data-Flow Analysis

Data-flow analysis (DFA) provides information about how the **execution of a program may manipulate its data**.

Many different problems can be formulated as **data-flow problems**, for example:

- Which assignments to variable  $v$  may influence a use of  $v$  at a certain program position?
- Is a variable  $v$  used on any path from a program position  $p$  to the exit node?
- The values of which expressions are available at program position  $p$ ?

Data-flow problems are stated in terms of

- **paths through the control-flow graph** and
- **properties of basic blocks**.

Data-flow analysis provides information for **global optimization**.

**Data-flow analysis does not know**

- which input values are provided at run-time,
- which branches are taken at run-time.

Its results are to be interpreted **pessimistic**

## Data-Flow Equations

A data-flow problem is stated as a **system of equations** for a control-flow graph.

System of Equations for **forward problems** (propagate information along control-flow edges):

Example **Reaching definitions**:

A definition  $d$  of a variable  $v$  reaches the begin of a block  $B$  if **there is a path** from  $d$  to  $B$  on which  $v$  is not assigned again.

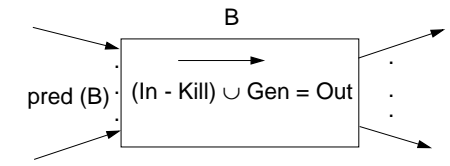
**In, Out, Gen, Kill** represent **analysis information**:

sets of statements,  
sets of variables,  
sets of expressions  
depending on the analysis problem

**2 equations for each basic block:**

$$\begin{aligned} \text{Out}(B) &= f_B(\text{In}(B)) \\ &= \text{Gen}(B) \cup (\text{In}(B) - \text{Kill}(B)) \end{aligned}$$

$$\text{In}(B) = \bigcap_{h \in \text{pred}(B)} \text{Out}(h)$$



In, Out **variables** of the system of equations for each block

Gen, Kill a pair of **constant sets** that characterize a block w.r.t. the DFA problem

$\Theta$  meet operator; e. g.  $\Theta = \cup$  for „reaching definitions“,  $\Theta = \cap$  for „available expressions“

## Specification of a DFA Problem

Specification of reaching definitions:

### 1. Description:

A definition  $d$  of a variable  $v$  reaches the begin of a block  $B$  if **there is a path** from  $d$  to  $B$  on which  $v$  is not assigned again.

### 2. It is a **forward problem**.

### 3. The **meet operator** is union.

### 4. The **analysis information** in the sets are assignments at certain program positions.

### 5. **Gen(B)**:

contains all definitions  $d: v = e$ ; in  $B$ , such that  $v$  is not defined after  $d$  in  $B$ .

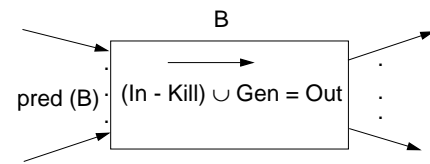
### 6. **Kill(B)**:

if  $v$  is assigned in  $B$ , then **Kill(B)** contains all definitions  $d: v = e$ ; of blocks different from  $B$ .

**2 equations for each basic block:**

$$\begin{aligned} \text{Out}(B) &= f_B(\text{In}(B)) \\ &= \text{Gen}(B) \cup (\text{In}(B) - \text{Kill}(B)) \end{aligned}$$

$$\text{In}(B) = \bigcap_{h \in \text{pred}(B)} \text{Out}(h)$$



## Variants of DFA Problems

### • **forward** problem:

DFA information flows **along the control flow**

In(B) is determined by Out(h) of the predecessor blocks

### **backward** problem (see C-2.23):

DFA information flows **against the control flow**

Out(B) is determined by In(h) of the successor blocks

### • **union** problem:

problem description: „there is a path“;

meet operator is  $\Theta = \cup$

solution: minimal sets that solve the equations

### **intersect** problem:

problem description: „for all paths“

meet operator is  $\Theta = \cap$

solution: maximal sets that solve the equations

### • **optimization information**: sets of certain statements, of variables, of expressions.

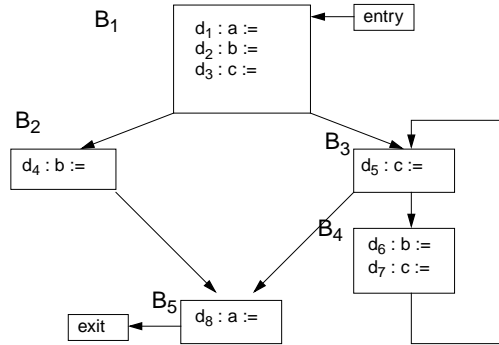
Further classes of DFA problems over general lattices instead of sets are not considered here.



### Example Reaching Definitions

**Gen (B):**  
contains all definitions  $d: v = e;$  in  $B$ , such that  $v$  is not defined after  $d$  in  $B$ .

**Kill (B):**  
contains all definitions  $d: v = e;$  in blocks different from  $B$ , such that  $B$  has a definition of  $v$ .



Description of DFA-Problem		
	Gen	Kill
<b>B1</b>	d1, d2, d3	d4, d5, d6, d7, d8
<b>B2</b>	d4	d2, d6
<b>B3</b>	d5	d3, d7
<b>B4</b>	d6, d7	d2, d3, d4, d5
<b>B5</b>	d8	d1

DFA-Solution		
	In	Out
	$\emptyset$	d1, d2, d3
<b>B2</b>	d1, d2, d3	d1, d3, d4
<b>B3</b>	d1, d2, d3, d6, d7	d1, d2, d5, d6
<b>B4</b>	d1, d2, d5, d6	d1, d6, d7
<b>B5</b>	d1, d2, d3, d4, d5, d6	d2, d3, d4, d5, d6, d8

### Iterative Solution of Data-Flow Equations

Input: the CFG; the sets Gen(B) and Kill(B) for each basic block B  
Output: the sets In(B) and Out(B)

**Algorithm:**

```

repeat
  stable := true;
  for all B ≠ entry { * }
  do begin
    for all V ∈ pred(B) do
      In(B) := In(B) ∩ Out(V);
    oldout := Out(B);
    Out(B) := Gen(B) ∪ (In(B) - Kill(B));
    stable := stable and Out(B) = oldout;
  end
until stable
            
```

**Initialization**

**Union:** empty sets

```

for all B do
  begin
    In(B) := ∅;
    Out(B) := Gen(B);
  end;
            
```

**Intersect:** full sets

```

for all B do
  begin
    In(B) := U;
    Out(B) :=
      Gen(B) ∪
      (U - Kill(B));
  end;
            
```

Complexity:  $O(n^3)$  with n number of basic blocks  
 $O(n^2)$  if  $|\text{pred}(B)| \leq k \ll n$  for all B

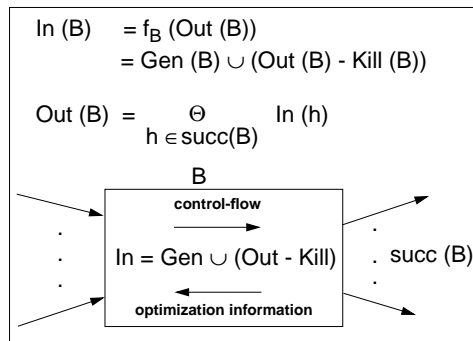
### Backward Problems

System of Equations for **backward problems** propagate information against control-flow edges:

2 equations for each basic block:

Example **Live variables**:

1. Description: Is variable  $v$  alive at a given point  $p$  in the program, i. e. **is there a path** from  $p$  to the exit where  $v$  is used but not defined before the use?
2. backward problem
3. optimization information: sets of variables
4. meet operator:  $\Theta = \cup$  union
5. Gen (B): variables that are used in B, but not defined before they are used there.
6. Kill (B): variables that are defined in B, but not used before they are defined there.



### Important Data-Flow Problems

1. **Reaching definitions:** A definition  $d$  of a variable  $v$  reaches the beginning of a block  $B$  if there is a path from  $d$  to  $B$  on which  $v$  is not assigned again.  
**DFA variant:** forward; union; set of assignments  
**Transformations:** use-def-chains, constant propagation, loop invariant computations
2. **Live variables:** Is variable  $v$  alive at a given point  $p$  in the program, i. e. there is a path from  $p$  to the exit where  $v$  is used but not defined before the use.  
**DFA variant:** backward; union; set of variables  
**Transformations:** eliminate redundant assignments
3. **Available expressions:** Is expression  $e$  computed on every path from the entry to a program position  $p$  and none of its variables is defined after the last computation before  $p$ .  
**DFA variant:** forward; intersect; set of expressions  
**Transformations:** eliminate redundant computations
4. **Copy propagation:** Is a copy assignment  $c: x = y$  redundant, i.e. on every path from  $c$  to a use of  $x$  there is no assignment to  $y$ ?  
**DFA variant:** forward; intersect; set of copy assignments  
**Transformations:** remove copy assignments and rename use
5. **Constant propagation:** Has variable  $x$  at position  $p$  a known value, i.e. on every path from the entry to  $p$  the last definition of  $x$  is an assignment of the same known value.  
**DFA variant:** forward; combine function; vector of values  
**Transformations:** substitution of variable uses by constants

## Algebraic Foundation of DFA

C-2.24a

DFA performs computations on a **lattice (dt. Verband)** of values, e. g. bit-vectors representing finite sets. It guarantees termination of computation and well-defined solutions. see [Muchnick, pp 223-228]

A **lattice L** is a set of values with two operations:  $\cap$  **meet** and  $\cup$  **join**

Required properties:

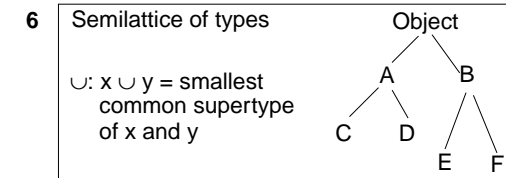
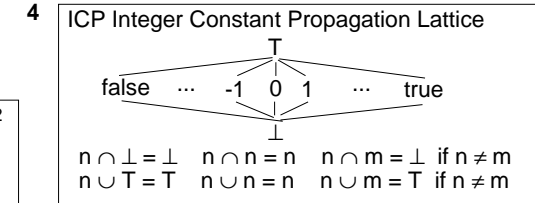
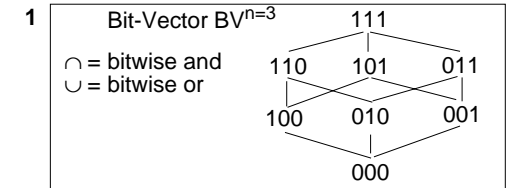
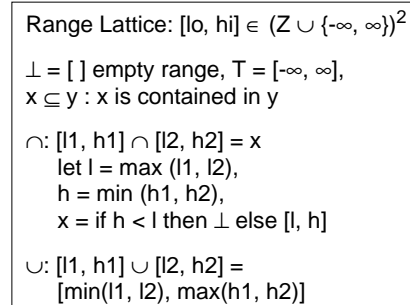
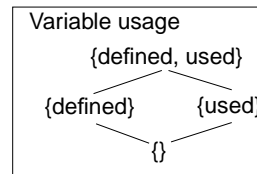
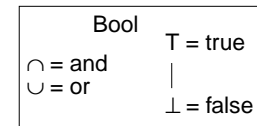
- closure:**  $x, y \in L$  implies  $x \cap y \in L, x \cup y \in L$
- commutativity:**  $x \cap y = y \cap x$  and  $x \cup y = y \cup x$
- associativity:**  $(x \cap y) \cap z = x \cap (y \cap z)$  and  $(x \cup y) \cup z = x \cup (y \cup z)$
- absorption:**  $x \cap (x \cup y) = x = x \cup (x \cap y)$
- unique elements **bottom**  $\perp$ , **top**  $T$ :  
 $x \cap \perp = \perp$  and  $x \cup T = T$

In most DFA problems only a **semilattice** is used with  $L, \cap, \perp$  or  $L, \cup, T$

**Partial order** defined by meet, defined by join:  
 $x \subseteq y: x \cap y = x$        $x \supseteq y: x \cup y = x$   
 (transitive, antisymmetric, reflexive)

## Some DFA Lattices

C-2.24b



## Monotone Functions Over Lattices

C-2.24c

The **effects of program constructs on DFA information** are described by functions over a suitable lattice,

e. g. the function for basic block  $B_3$  on C-2.22:

$$f_3(\langle x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8 \rangle) = \langle x_1 \ x_2 \ 0 \ x_4 \ 1 \ x_6 \ 0 \ x_8 \rangle \in BV^8$$

### Gen-Kill pair encoded as function

$f: L \rightarrow L$  is a **monotone function** over the lattice L if  
 $\forall x, y \in L: x \subseteq y \Rightarrow f(x) \subseteq f(y)$

**Finite height** of the lattice and **monotonicity** of the functions guarantee **termination** of the algorithms.

**Fixed points**  $z$  of the function  $f$ , with  $f(z) = z$ , is a solution of the set of DFA equations.

**MOP: Meet over all paths** solution is desired, i. e. the „best“ with respect to L

**MFP: Maximum fixed point** is computed by algorithms, if functions are monotone

If the functions  $f$  are additionally **distributive**, then **MFP = MOP**.

$f: L \rightarrow L$  is a **distributive function** over the lattice L if  
 $\forall x, y \in L: f(x \cap y) = f(x) \cap f(y)$

## Variants of DFA Algorithms

C-2.26

### Heuristic improvement:

Goal: propagate changes in the In and Out sets as fast as possible.  
 Technique: visit CFG nodes in topological order in the outer for-loop {\*}.  
 Then the number of iterations of the outer repeat-loop is only determined by back edges in the CFG

### Algorithm for backward problems:

Exchange In and Out sets symmetrically in the algorithm of C-2.22b.  
 The nodes should be visited in topological order as if the directions of edges were flipped.

### Hierarchical algorithms, interval analysis:

Regions of the CFG are considered nodes of a CFG on a higher level.  
 That abstraction is recursively applied until a single root node is reached.  
 The Gen, Kill sets are combined in upward direction;  
 the In, Out sets are refined downward.

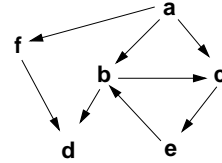
## Program Analysis: Call Graph (context-insensitive)

C-2.27

**Nodes:** defined functions

**Arc**  $g \rightarrow h$ : function  $g$  contains a call  $h()$ ,  
i. e. a call  $g()$  **may** cause the execution of a call  $h()$

```
void a () { ...b()...c()...f()... }
void b () { ...d()...c()... }
void c () { ...e()... }
void d () { ... }
void e () { ...v++;...b()... }
void f () { ...d()... }
```



**Analysis of structure:**  
b, c, e are recursive;  
a, d, f are non-recursive

### Propagation of properties:

assume a call  $e()$  may **modify a global variable**  $v$   
then calls  $a()$ ,  $b()$ ,  $c()$  may indirectly cause modification of  $v$

```
v = f(); cnt = 0; while(...){...b(); cnt += v;}
```

© 2013 bei Prof. Dr. Uwe Kastens

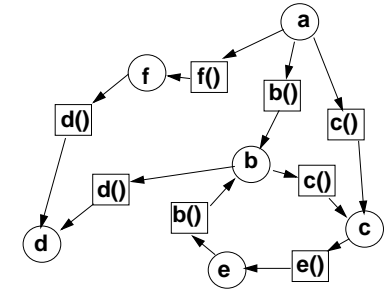
## Program Analysis: Call Graph (context-sensitive)

C-2.27a

**Nodes:** defined functions and calls (bipartite)

**Arc**  $g \rightarrow h$ : function  $g$  contains a call  $h()$ , i.e. a call  $g()$  **may** cause the execution of a call  $h()$   
or call  $g()$  leads to function  $g$

```
void a () { ...b()...c()...f()... }
void b () { ...d()...c()... }
void c () { ...e()... }
void d () { ... }
void e () { ...v++;...b()... }
void f () { ...d()... }
```



**Calls of the same function in different contexts** are distinguished by **different nodes**, e.g. the call of  $c$  in  $a$  and in  $b$ .

Analysis can be **more precise** in that aspect.

© 2013 bei Prof. Dr. Uwe Kastens

## Calls Using Function Variables

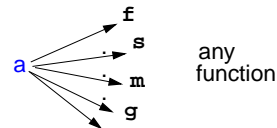
C-2.28

Contents of **function variables** is assigned at run-time.

Static analysis does not know (precisely) which function is called.

**Call graph** has to assume that **any function may be called**.

```
void a()
{ ...(*h)(0.3, 27)... }
```



**Analysis for a better approximation**  
of potential callees:

only those functions which

1. **fit to the type** of  $h$
2. **are assigned** somewhere in the program
3. can be derived from the **reaching definitions** at the call

```
void m (int j) { ... }
void g (float x, int i) { ... }
...k = m;... f(g); ...
void a()
{ void (*h)(float,int) = g;
  ...
  if(...) h = s;
  ...(*h)(0.3, 27)...
}
```

© 2006 bei Prof. Dr. Uwe Kastens

## Analysis of Object-Oriented Programs

C-2.29

Aspects specific for object-oriented analysis:

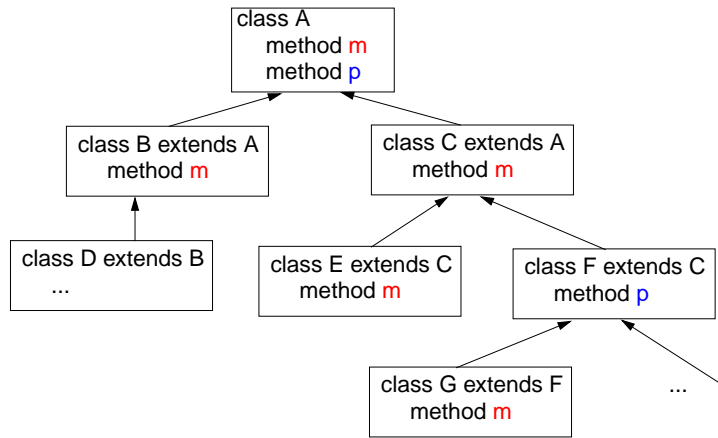
1. **hierarchy of classes and interfaces**  
specifies a complex **system of subtypes**
2. **hierarchy of classes and interfaces**  
specifies **inheritance and overriding** relation for methods
3. **dynamic method binding**  
for method calls  $v.m(\dots)$  the **callee is determined at run-time**  
good object-oriented style relies on that feature
4. **many small methods** are typical object-oriented style
5. **library use and reuse of modules**  
complete program contains many **unused classes and methods**

**Static predictions for dynamically bound method calls**  
are essential for most analyses

© 2004 bei Prof. Dr. Uwe Kastens

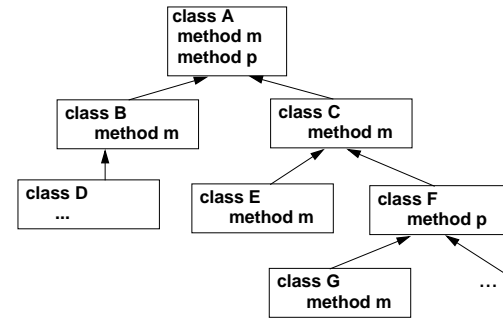
### Class Hierarchy Graph

**Node:** class or interface  
**Arc a -> b:** a is subclass of b or a implements interface b

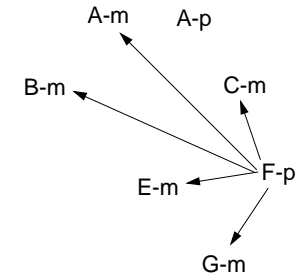


### Object-Oriented Call Graph

**Node:** implemented method, identified by class name, method name: X-a  
**Arc X-a -> Y-b:** method X-a contains a call v.b(...) that may be bound to Y-b



Call graph for F-p containing v.m(...)



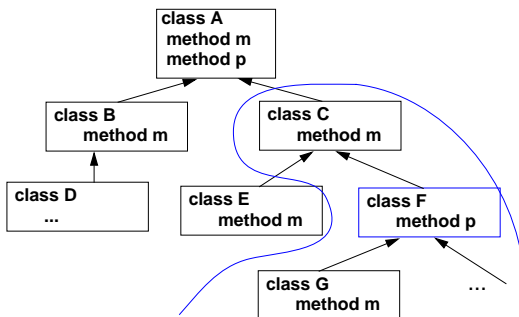
Call graph: **any method m** may be bound to that call in F-p (compare to function variables) analysis yields better approximations

### Call Graphs Constructed by Class Hierarchy Analysis (CHA)

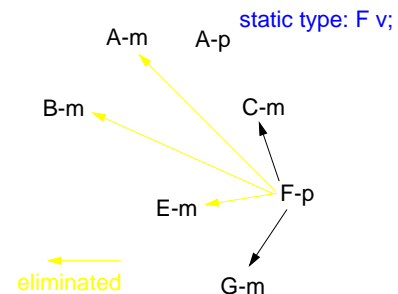
The call graph is reduced to a set of **reachable methods** using the **class hierarchy** and the **static type of the receiver** expression in the call:

If a method F-p is **reachable** and if it contains a **dynamically bound call v.m(...)** and **T is the static type of v**,

then every method **m** that is **inherited by T** or by a **subtype of T** is **also reachable**, and arcs go from F-p to them.



Call graph for F-p containing v.m(...)



### Refined Approximations for Call Graph Construction

**Class Hierarchy Analysis (CHA):** (see C-2.32)

**Rapid Type Analysis (RTA):**

As CHA, but only methods of those classes C are considered which are instantiated (`new C()`) in a reachable method.

**Reaching Type Analysis:**

Approximations of run-time types is propagated through a graph: nodes represent variables, arcs represent copy assignments.

**Declared Type Analysis:**

one node T represents all variables declared to have type T

**Variable Type Analysis:**

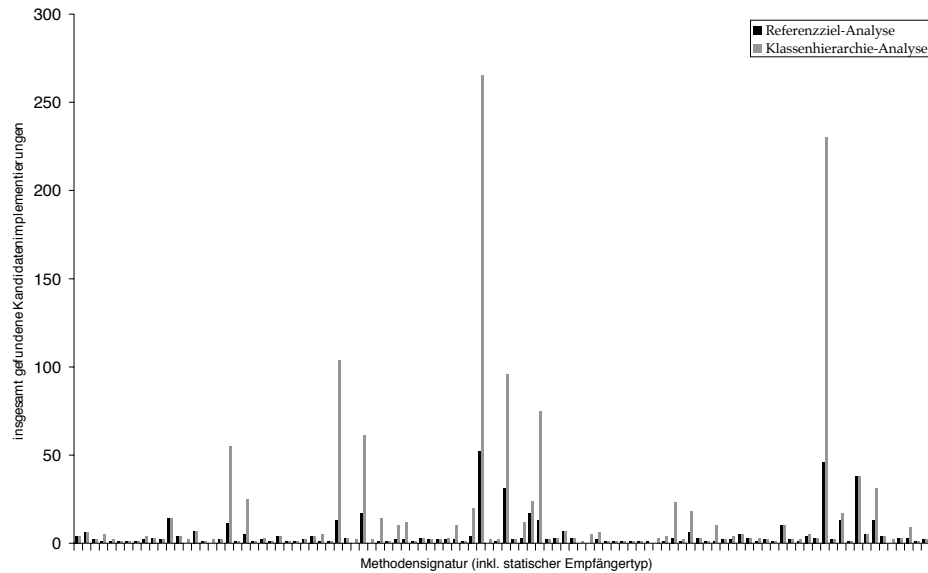
one node V represents a single variable

**Points-to Analysis:**

Information on object identities is propagated through the control-flow graph

## Results of Analysis of Dynamically Bound Calls

C-2.34



© 2002 bei Prof. Dr. Uwe Kastens

## Modules of a Toolset for Program Analysis

C-2.35

analysis module	purpose	category
ClassMemberVisibility	examines visibility levels of declarations	visualization
MethodSizeStatistics	examines length of method implementations in bytecode operations and frequency of different bytecode operations	
ExternalEntities	histogram of references to program entities that reside outside a group of classes	
InheritanceBoundary	histogram of lowest superclass outside a group of classes	
SimpleSetterGetter	recognizes simple access methods with bytecode patterns	auxiliary analysis
MethodInspector	decomposes the raw bytecode array of a method implementation into a list of instruction objects	
ControlFlow	builds a control flow graph for method implementations	fundamental analyses
Dominator	constructs the dominator tree for a control flow graph	
Loop	uses the dominator tree to augment the control flow graph with loop and loop nesting information	
InstrDefUse	models operand accesses for each bytecode instruction	
LocalDefUse	builds intraprocedural def/use chains	
LifeSpan	analyzes liveness of local variables and stack locations	analysis of incomplete programs
DefUseTypeInfo	infers type information for operand accesses	
Hierarchy	class hierarchy analysis based on a horizontal slice of the hierarchy	
PreciseCallGraph	builds call graph based on inferred type information, copes with incomplete class hierarchy	
ParamEscape	transitively traces propagation of actual parameters in a method call (escape = leaves analyzed library)	
ReadWriteFields	transitive liveness and access analysis for instance fields accessed by a method call	

Table 0-1. Analysis plug-ins in our framework

[ Michael Thies: *Combining Static Analysis of Java Libraries with Dynamic Optimization*, Dissertation, Shaker Verlag, April 2001]

© 2002 bei Prof. Dr. Uwe Kastens

## 3. Code Generation

C-3.1

**Input:** Program in intermediate language

### Tasks:

- Storage mapping    properties of program objects (size, address) in the definition module
- Code selection     generate instruction sequence, optimizing selection
- Register allocation    use of registers for intermediate results and for variables

**Output:** abstract machine program, stored in a data structure

### Design of code generation:

- analyze **properties of the target processor**
- plan **storage mapping**
- design at least one **instruction sequence** for each operation of the intermediate language

### Implementation of code generation:

- Storage mapping: a traversal through the program and the definition module computes sizes and addresses of storage objects
- Code selection: use a generator for pattern matching in trees
- Register allocation: methods for expression trees, basic blocks, and for CFGs

© 2002 bei Prof. Dr. Uwe Kastens

## 3.1 Storage Mapping

C-3.2

### Objective:

for each storable program object compute storage class, relative address, size

### Implementation:

use properties in the definition module, traverse defined program objects

### Design the use of storage areas:

- code storage    program code
- global data    to be linked for all compilation units
- run-time stack    activation records for function calls
- heap            storage for dynamically allocated objects, garbage collection
- registers for    addressing of storage areas (e. g. stack pointer) function results, arguments local variables, intermediate results (**register allocation**)

### Design the mapping of data types (next slides)

### Design activation records and translation of function calls (next section)

© 2002 bei Prof. Dr. Uwe Kastens

## Storage Mapping for Data Types

C-3.3

### Basic types

arithmetic, boolean, character types

match language requirements and machine properties:  
data format, available instructions,  
size and alignment in memory

### Structured types

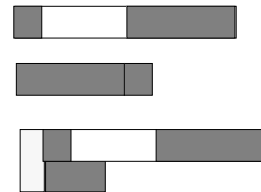
for each type representation in memory and  
code sequences for operations,  
e. g. assignment, selection, ...

**record** relative address and  
alignment of components;  
reorder components for optimization

**union** storage overlay,  
tag field for discriminated union

**set** bit vectors, set operations

for **arrays** and **functions** see next slides



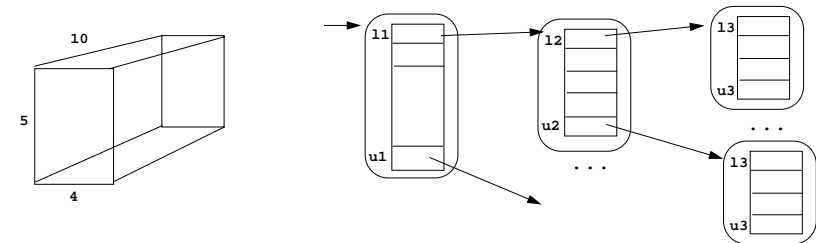
## Array Implementation: Pointer Trees

C-3.4

An n-dimensional array

**a: array[11..u1, 12..u2, ..., 1n..un] of real;**

is implemented by a **tree of linear arrays**;  
n-1 levels of pointer arrays and data arrays on the n-th level



Each single array can be allocated separately, dynamically; scattered in memory  
In **Java arrays** are implemented this way.

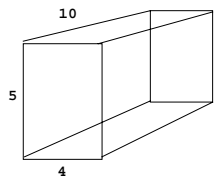
## Array Implementation: Contiguous Storage

C-3.5

An n-dimensional array

**a: array[11..u1, 12..u2, ..., 1n..un] of real;**

is mapped to **one contiguous storage area**  
**linearized in row-major order:**



linear storage map of array a onto byte-array **store** from index **start**:  
number of elements  $elno = st1 * st2 * \dots * stn$   
i-th index stride  $sti = ui - li + 1$   
element size in bytes  $elsz$

Index map of **a[i1, i2, ..., in]**:

$store[start + ((i1-l1)*st2 + (i2-l2))*st3 + \dots * stn + (in-ln)*elsz]$   
 $store[const + (i1*st2 + i2)*st3 + \dots * stn + in)*elsz]$

## Functions as Data Objects

C-3.6

Functions may occur **as data objects**:

- variables
- parameters
- function results
- lambda expressions  
(in functional languages)

Functions that are defined on the  
**outermost program level** (non-nested)  
can be implemented by just the  
**address of the code**.

Functions that are **defined in nested structures** have to be  
implemented by a **pair: (closure, code)**

The **closure** contains all **bindings** of names to variables or values that  
are valid when the **function definition is executed**.

In **run-time stack** implementations the  
**closure is a sequence of activation records on the static  
predecessor chain**.

### 3.2 Run-Time Stack Activation Records

**Run-time stack** contains one **activation record** for each active function call.

**Activation record:**  
provides storage for the data of a function call.

**dynamic link:**  
link from callee to caller,  
to the preceding record on the stack

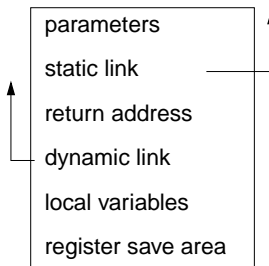
**static link:**  
**link from callee c to the record s where c is defined**  
s is a call of a function which contains the definition of the function, the call of which created c.

**Variables of surrounding functions** are accessed via the static predecessor chain.

Only relevant for languages which allow **nested functions**, classes, objects.

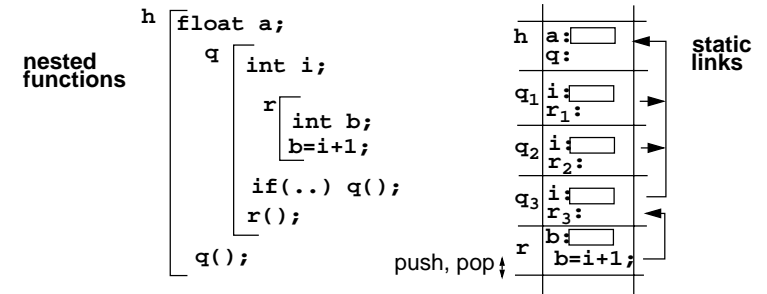
**closure of a function call:**  
the **activation records on the static predecessor chain**

**activation record:**



### Example for a Run-Time Stack

**Run-time stack:**  
A call creates an activation record and pushes it onto the stack.  
It is popped on termination of the call.



The **static link** points to the activation record where the called function is defined, e. g. r<sub>3</sub> in q<sub>3</sub>

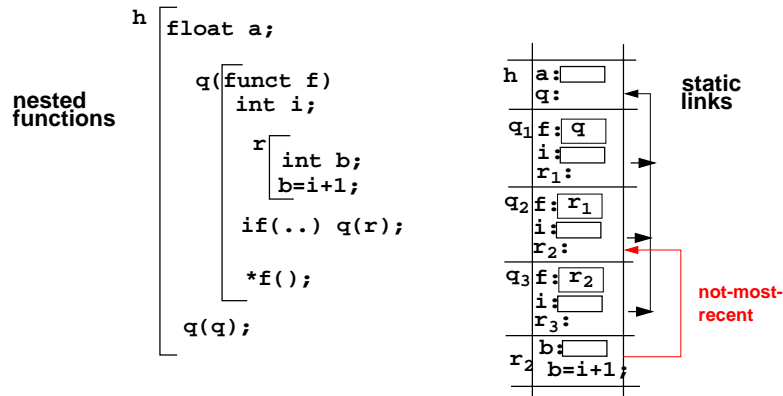
Optimization: activation records of **non-recursive functions** may be allocated statically.  
Languages without recursive functions (FORTRAN) do not need a run-time stack.

Parallel processes, threads, and coroutines need a **separate run-time stack** each.

### Not-Most-Recent Property

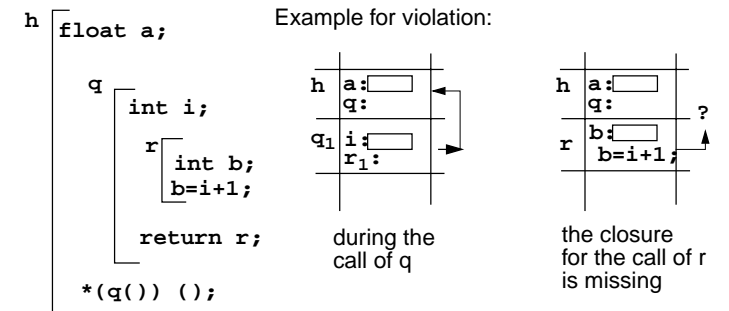
The **static link** of an activation record c for a function r points to an activation record d for a function q where r is defined in.  
If there are activation records for q on the stack, that are more recently created than d, the **static link to d is not-most-recent**.

That effect can be achieved by using functional parameters or variables.  
Example:



### Closures on Run-Time Stacks

Function calls can be implemented by a run-time stack if the **closure of a function is still on the run-time stack when the function is called.**



**Language conditions** to guarantee run-time stack discipline:

Pascal: functions not allowed as function results, or variables

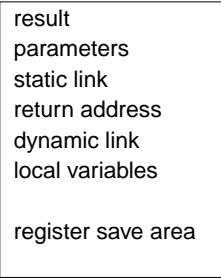
C: no nested functions

Modula-2: nested functions not allowed as values of variables

**Functional languages** maintain activation records on the heap instead of the run-time stack

### Activation Records and Call Code

**activation record:**



**call code**

push parameter values  
 push static link  
 subroutine jump

**function code**

push dynamic link  
 stack register := top of stack  
 increment top of stack  
 for local variables  
 save registers  
 ...  
 function body  
 ...  
 restore registers  
 deallocate local variables  
 pop stack register  
 return jump

pop static link  
 pop parameter area  
 use and pop result

### 3.3 Code Sequences for Control Statements

A **code sequence** defines how a **control statement** is transformed into jumps and labels.

**Notation of the Code constructs:**

- Code (S)** generate code for statements *s*
- Code (C, true, M)** generate code for condition *c* such that it branches to *M* if *c* is true, otherwise control continues without branching
- Code (A, Ri)** generate code for expression *A* such that the result is in register *Ri*

**Code sequence for if-else statement:**

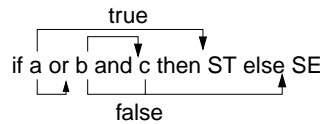
```

if (cond) ST; else SE;:

    Code (cond, false, M1)
    Code (ST)
    goto M2
M1: Code (SE)
M2:
    
```

### Short Circuit Translation of Boolean Expressions

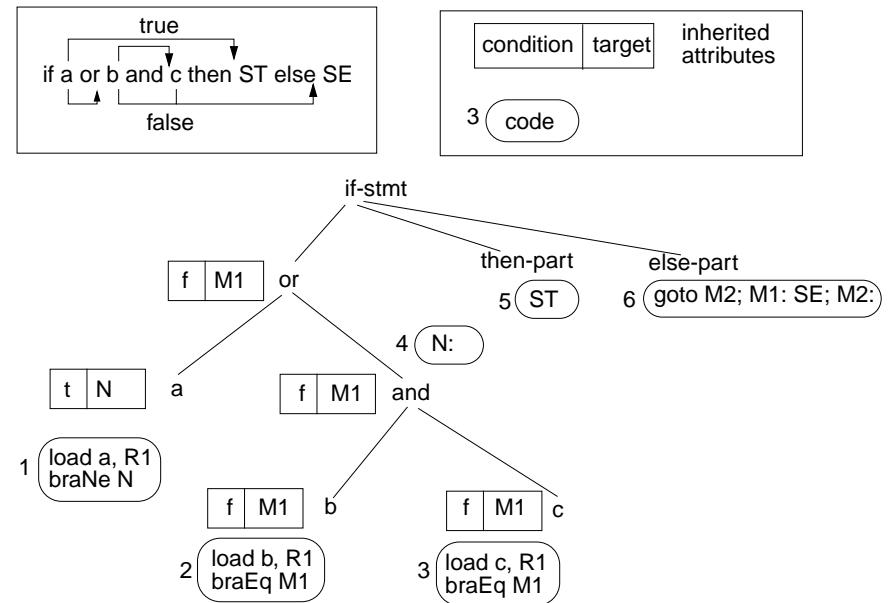
**Boolean expressions** are translated into **sequences of conditional branches**. Operands are evaluated from left to right until the result is determined.



2 code sequences for each operator; applied to condition tree on a top-down traversal:

- Code (A and B, true, M):** Code (A, false, N)  
Code (B, true, M)  
N:
- Code (A and B, false, M):** Code (A, false, M)  
Code (B, false, M)
- Code (A or B, true, M):** Code (A, true, M)  
Code (B, true, M)
- Code (A or B, false, M):** Code (A, true, N)  
Code (B, false, M)  
N:
- Code (not A, X, M):** Code (A, not X, M)
- Code (A < B, true, M):** Code (A, Ri);  
Code (B, Rj)  
cmp Ri, Rj  
braLt M
- Code (A < B, false, M):** Code (A, Ri);  
Code (B, Rj)  
cmp Ri, Rj  
braGe M
- Code for a leaf:** conditional jump

### Example for Short Circuit Translation





## Code Sequences for Loops

C-3.15

### While-loop variant 1:

```
while (Condition) Body
    M1: Code (Condition, false, M2)
        Code (Body)
        goto M1
    M2:
```

### While-loop variant 2:

```
while (Condition) Body
    goto M2
    M1: Code (Body)
    M2: Code (Condition, true, M1)
```

### Pascal for-loop unsafe variant:

```
for i:= Init to Final do Body
    i = Init
    L: if (i>Final) goto M
        Code (Body)
        i++
        goto L
    M:
```

### Pascal for-loop safe variant:

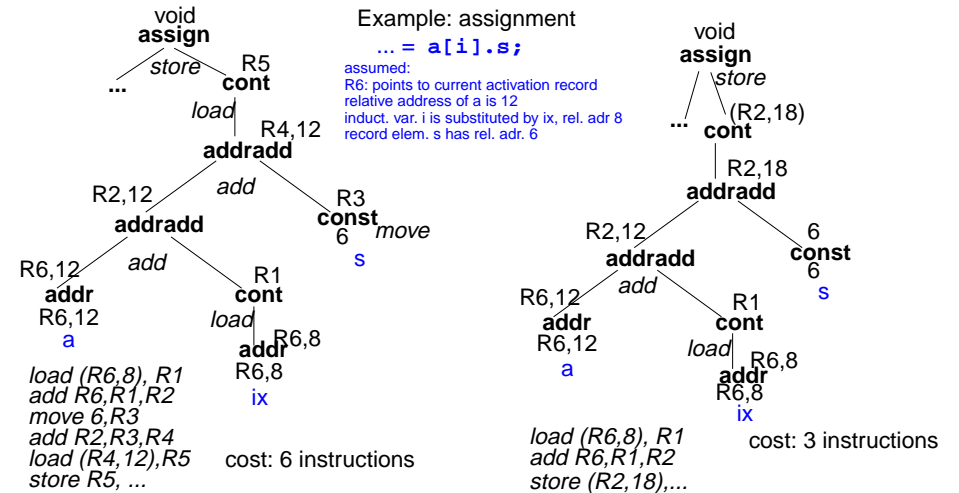
```
for i:= Init to Final do Body
    if (Init==minint) goto L
    i = Init - 1
    goto N
    L: Code (Body)
    N: if (i>= Final) goto M
        i++
        goto L
    M:
```

© 2007 bei Prof. Dr. Uwe Kastens

## 3.4 Code Selection

C-3.16

- Given: target tree in intermediate language.
- Optimizing selection:** Select patterns that translate single nodes or small subtrees into machine instructions; cover the whole tree with as few instructions as possible.
- Method: **Tree pattern matching**, several techniques



© 2011 bei Prof. Dr. Uwe Kastens

## Selection Technique: Value Descriptors

C-3.17

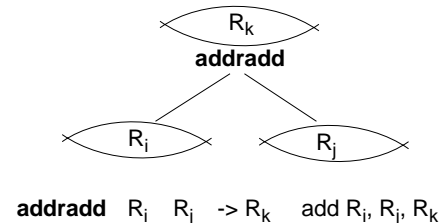
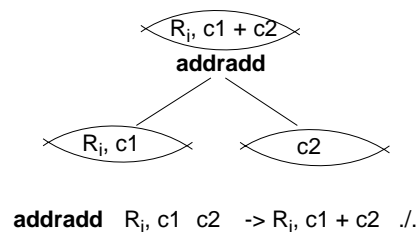
Intermediate language **tree node operators**;  
 e.g.:

**addr** address of variable  
**const** constant value  
**cont** load contents of address  
**addradd** address + value

**Value descriptors** state how/where the  
 value of a tree node is represented, e. g.

**R<sub>i</sub>** value in register R<sub>i</sub>  
**c** constant value c  
**R<sub>i</sub>,c** address R<sub>i</sub> + c  
**(adr)** contents at the address adr

alternative **translation patterns** to be selected context dependend:



© 2007 bei Prof. Dr. Uwe Kastens

## Example for a Set of Translation Patterns

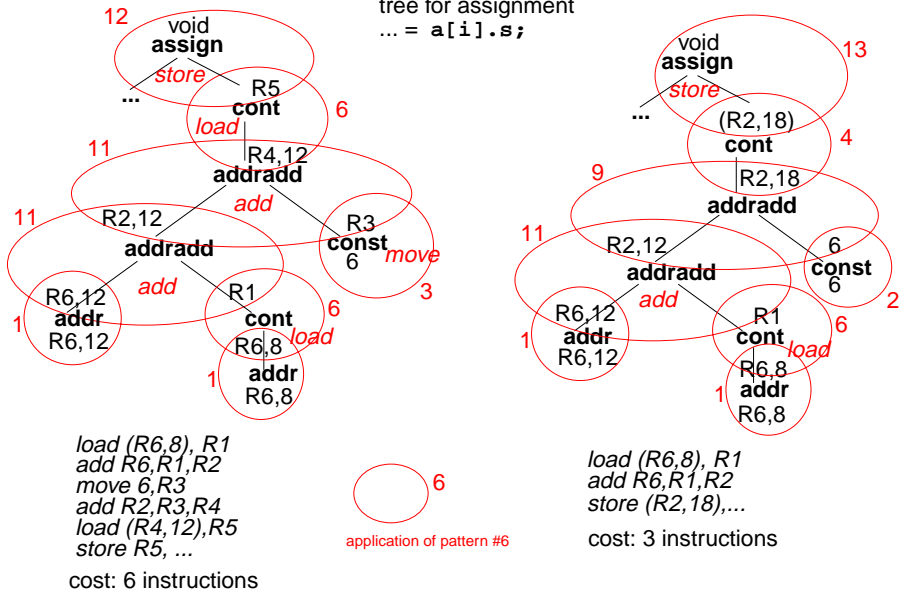
C-3.18

#	operator	operands	result	code
1	addr	R <sub>i</sub> , c	-> R <sub>i</sub> ,c	./.
2	const	c	-> c	./.
3	const	c	-> R <sub>i</sub>	move c, R <sub>i</sub>
4	cont	R <sub>i</sub> , c	-> (R <sub>i</sub> , c)	./.
5	cont	R <sub>i</sub>	-> (R <sub>i</sub> )	./.
6	cont	R <sub>i</sub> , c	-> R <sub>j</sub>	load (R <sub>i</sub> , c), R <sub>j</sub>
7	cont	R <sub>i</sub>	-> R <sub>j</sub>	load (R <sub>i</sub> ), R <sub>j</sub>
8	addradd	R <sub>i</sub> c	-> R <sub>i</sub> , c	./.
9	addradd	R <sub>i</sub> , c1 c2	-> R <sub>i</sub> , c1 + c2	./.
10	addradd	R <sub>i</sub> R <sub>j</sub>	-> R <sub>k</sub>	add R <sub>i</sub> , R <sub>j</sub> , R <sub>k</sub>
11	addradd	R <sub>i</sub> , c R <sub>j</sub>	-> R <sub>k</sub> , c	add R <sub>i</sub> , R <sub>j</sub> , R <sub>k</sub>
12	assign	R <sub>i</sub> R <sub>j</sub>	-> void	store R <sub>j</sub> , R <sub>i</sub>
13	assign	R <sub>i</sub> (R <sub>j</sub> , c)	-> void	store (R <sub>j</sub> ,c), R <sub>i</sub>
14	assign	R <sub>i</sub> ,c R <sub>j</sub>	-> void	store R <sub>j</sub> , R <sub>i</sub> ,c

© 2011 bei Prof. Dr. Uwe Kastens

### Tree Covered with Translation Patterns

tree for assignment  
... = a[i].s;



### Pattern Selection

#### Pass 1 bottom-up:

Annotate the nodes with sets of pairs  
{ (v, c) | v is a kind of value descriptor that an applicable pattern yields, c are the accumulated subtree costs}

If (v, c1), (v, c2) keep only the cheaper pair.

#### Pass 2 top-down:

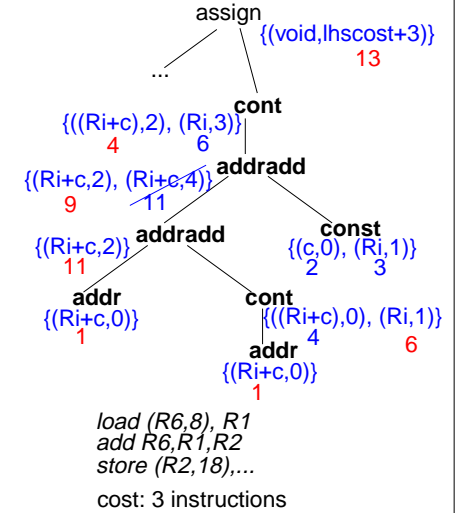
Select for each node the cheapest pattern, that fits to the selection made above.

#### Pass 3 bottom-up:

Emit code.

Improved technique:

relative costs per sets =>  
finite number of potential sets  
integer encoding of the sets at generation time



### Pattern Matching in Trees: Bottom-up Rewrite

#### Bottom-up Rewrite Systems (BURS) :

a general approach of the pattern matching method:

Specification in form of tree patterns, similar to C-3.18 - C-3.20

Set of patterns is **analyzed at generation** time.

Generator produces a **tree automaton** with a finite set of states.

On the bottom-up traversal it annotates each tree node with a **set of states**:

those selection decisions which may lead to an optimal solution.

Decisions are made on the base of the **costs of subtrees** rather than costs of nodes.

Generator: BURG

### Tree Pattern Matching by Parsing

The tree is represented in prefix form.

Translation patterns are specified by tuples (CFG production, code, cost), Value descriptors are the nonterminals of the grammar, e. g.

8	RegConst ::= <b>addradd</b> Reg Const	nop	0
11	RegConst ::= <b>addradd</b> RegConst Reg	add R <sub>i</sub> , R <sub>j</sub> , R <sub>k</sub>	1

Deeper patterns allow for more effective optimization:

Void ::= <b>assign</b> RegConst <b>addradd</b> Reg Const	store (R <sub>i</sub> , c1),(R <sub>j</sub> , c2)	1
--	---	---

Parsing for an ambiguous CFG:

application of a production is decided on the base of the production costs rather than the accumulated subtree costs!

Technique „Graham, Glanville“  
Generators: GG, GGSS

## 4 Register Allocation

C-4.1

### Use of registers:

1. intermediate **results of expression evaluation**
2. reused results of expression evaluation (CSE)
3. contents of frequently used **variables**
4. **parameters** of functions, **function result** (cf. register windowing)
5. stack pointer, **frame pointer**, heap pointer, ...

**Number of registers is limited** - for each register class: address, integer, floating point

### Specific allocation methods for different context ranges:

- 4.1 expression trees (Sethi, Ullman)
- 4.2 basic blocks (Belady)
- 4.3 control flow graphs (graph coloring)

### Register allocation aims at reduction of

- number of memory accesses
- spill code, i. e. instructions that store and reload the contents of registers

**Symbolic registers:** allocate a new symbolic register to each value assignment (single assignment, no re-writing); defer allocation of real registers to a later phase.

© 2013 bei Prof. Dr. Uwe Kastens

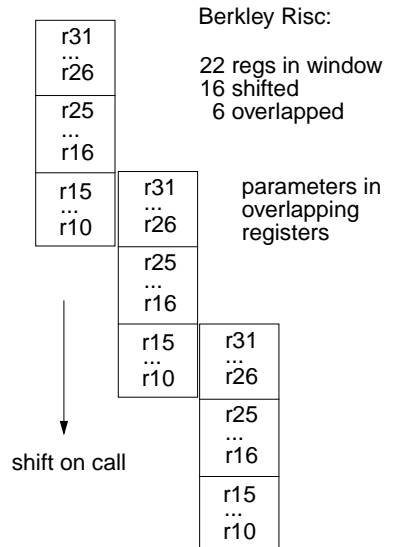
## Register Windowing

C-4.2

### Register windowing:

- Fast storage of the processor is accessed through a window.
- The  $n$  elements of the window are used as registers in instructions.
- On a call the window is shifted by  $m < n$  registers.
- Overlapping registers can be used under different names from both the caller and the callee.
- Parameters are passed without copying.
- Storage is organized in a ring; 4-8 windows; saved and restored as needed

Typical for Risc processors, e.g. Berkley RISC, SPARC

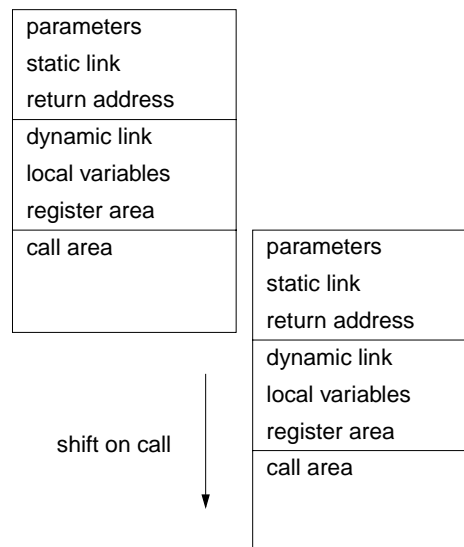


© 2002 bei Prof. Dr. Uwe Kastens

## Activation Records in Register Windows

C-4.3

- **Parameters** are passed in overlap area **without copying**.
- **Registers need not be saved** explicitly.
- If **window is too small** for an activation record, the remainder is allocated on the **run-time stack**; pointer to it in window.



© 2002 bei Prof. Dr. Uwe Kastens

## 4.1 Register Allocation for Expression Trees

C-4.4

### Problem:

Generate code for **expression** evaluation.  
**Intermediate results** are stored in registers.  
Not enough registers:  
**spill code** saves and restores.

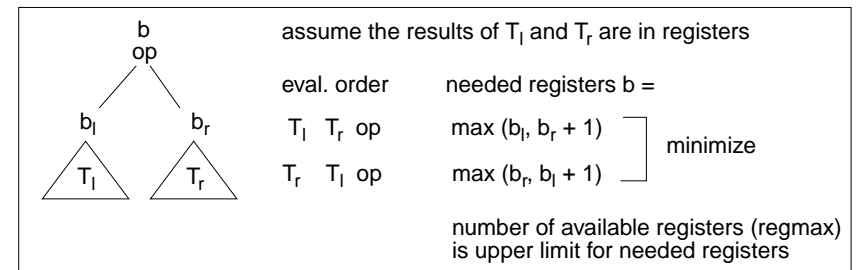
### Goal:

Minimize amount of spillcode.  
see C-4.5a for optimality condition

### Basic idea (Sethi, Ullman):

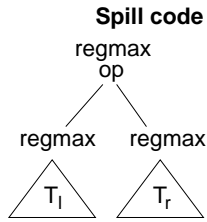
For each subtree minimize the **number of needed registers**:

evaluate **first the subtree that needs most** registers



© 2006 bei Prof. Dr. Uwe Kastens

### Expression Tree Attribution



Implementation by attribution of trees:

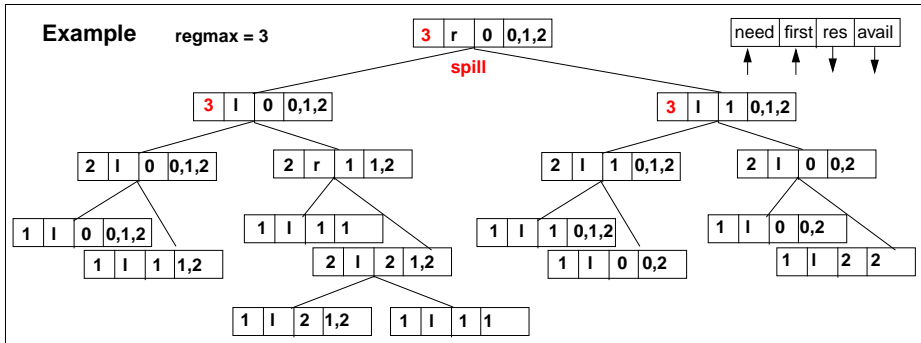
**Phase 1** bottom-up:  
needed registers, evaluation order

**Phase 2** top-down:  
allocate registers

**Phase 3** bottom-up:  
compose code in evaluation order

Code ( $T_r$ )  
store  $R_r, h$   
Code ( $T_l$ )  
load  $h, R_r$   
op  $R_r, R_l$

load  $h, R_r$  is not needed if  $h$  can be a memory operand in op  $h, R_l$



### Contiguous code vs. optimal code

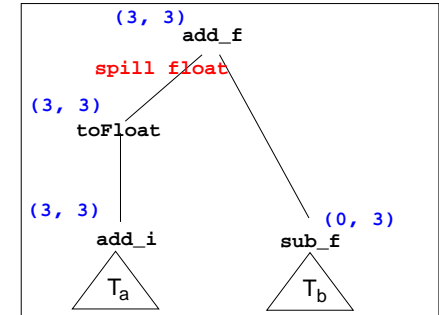
The method assumes that the **code for every subtree is contiguous**.  
(I.e. there is no interleaving between the code of any two disjoint subtrees.)

The **method is optimal** for a certain **configuration of registers and operations**, iff every **optimal evaluation code** can be arranged to be **contiguous**.

Counter example:

Registers: 3 int and 3 float  
Register need: (i, f) from (0, 0) to (3, 3)

Operations: int- and float- arithmetic,  
toFloat (widening)



register use: (3, 3) (1, 0) (0, 1) (0, 0) (0, 3) (0, 1) (0, 2) (0, 1)

**contiguous:** T<sub>a</sub> add\_i toFloat **spill float** T<sub>b</sub> sub\_f **load\_f** add\_f

**optimal:** T<sub>a</sub> add\_i T<sub>b</sub> sub\_f toFloat add\_f

register use: (3, 3) (1, 0) (1, 3) (1, 1) (1, 2) (0, 1)

### 4.2 Register Allocation for Basic Blocks by Life-Time Analysis

Lifetimes of values in a basic block are used to minimize the number of registers needed.

**1st Pass:** Determine the **life-times** of values: from the definition to the last use (there may be several uses!).

**Life-times are represented by intervals in a graph**

**cut of the graph** = number of **registers needed** at that point

**at the end of 1st pass:**

maximal cut = number of register needed for the basic block

allocate registers **in the graph**:

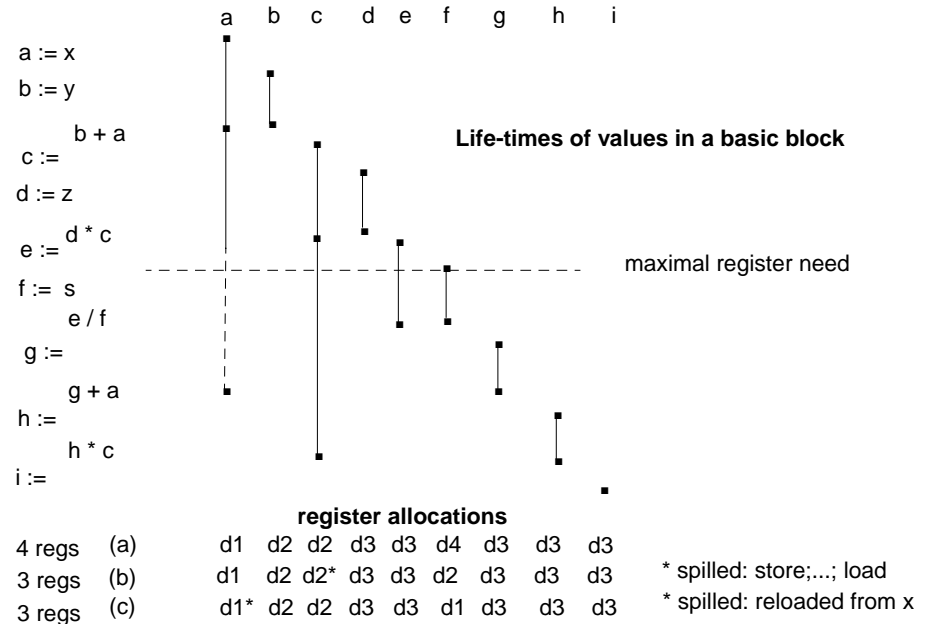
In case of shortage of registers: select values to be **spilled**; **criteria**:

- a **value that is already in memory** - store instruction is saved
- the **value that is latest used again**

**2nd Pass:** allocate registers **in the instructions**; evaluation order remains unchanged

The technique has been presented originally 1966 by **Belady** as a **paging technique for storage allocation**.

### Example for Belady's Technique



### 4.3 Register Allocation by Graph Coloring

C-4.8

Definitions and uses of variables in control-flow graphs for **function bodies** are analyzed (DFA). Conflicting life-times are modelled. Presented by **Chaitin**.

#### Construct an interference graph:

- Nodes:** Variables that are candidates for being kept in registers
- Edge {a, b}:** **Life-times** of variables a and b overlap => a, b have to be kept in different registers

Life-times for CFGs are determined by **data-flow analysis**.

#### Graph is „colored“ with register numbers.

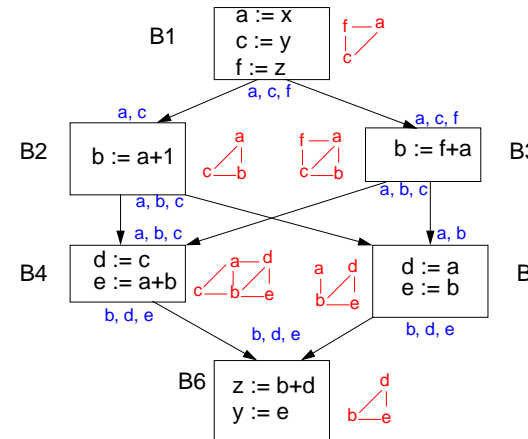
- NP complete problem; **heuristic technique** for coloring with k colors (registers):
  - eliminate nodes of degree < k (and its edges)
  - if the graph is finally empty:
    - graph can be colored with k colors
    - assign colors to nodes in reverse order of elimination
  - else
    - graph can not be colored this way
    - select a node for spilling
    - repeat the algorithm without that node

© 2011 bei Prof. Dr. Uwe Kastens

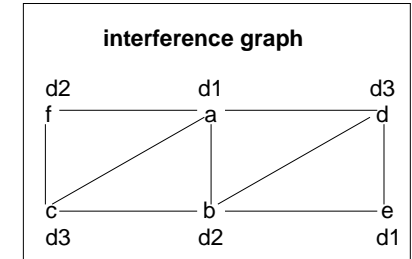
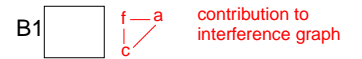
### Example for Graph Coloring

C-4.9

#### CFG with definitions and uses of variables



variables in memory: x, y, z  
 variables considered for register alloc.: a, b, c, d, e, f  
 results of live variable analysis: b, d, e

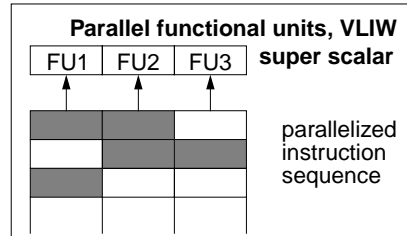


© 2013 bei Prof. Dr. Uwe Kastens

### 5 Code Parallelization

C-5.1

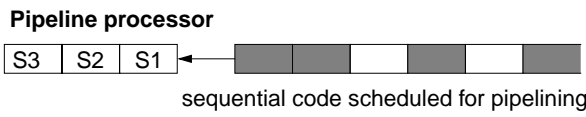
Processor with **instruction level parallelism (ILP)** executes several instructions in parallel.



- Classes of processors and parallelism: VLIW, super scalar, Pipelined processors, Data parallel processors

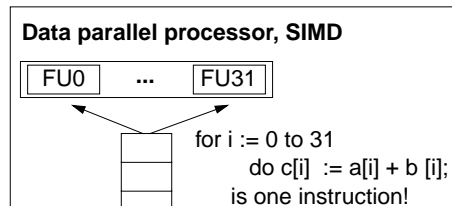
Compiler **analyzes sequential programs** to exhibit potential parallelism on instruction level;

model **dependencies** between computations



Compiler arranges instructions for shortest execution time: **instruction scheduling**

Compiler **analyzes loops** to execute them in parallel **loop transformation** **array transformation**



© 2009 bei Prof. Dr. Uwe Kastens

### 5.1 Instruction Scheduling Data Dependence Graph

C-5.2

Exhibit potential **fine-grained parallelism** among operations. Sequential code is over-specified!

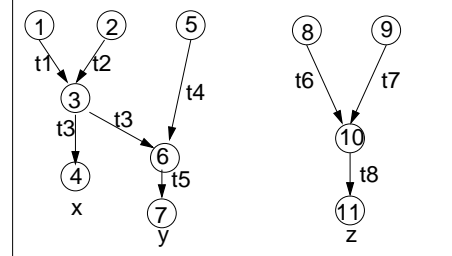
**Data dependence graph (DDG)** for a basic block:

- Node:** operation;
- Edge** a -> b: operation b uses the result of operation a

#### Example for a basic block:

- t1 := a
- t2 := b
- t3 := t1 + t2
- x := t3
- t4 := c
- t5 := t3 + t4
- y := t5
- t6 := d
- t7 := e
- t8 := t6 + t7
- z := t8

#### data dependence graph



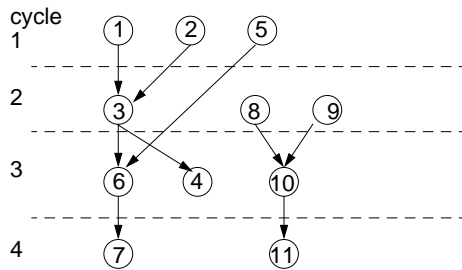
**ti** are **symbolic registers**, store intermediate results, obey single assignment rule

© 2002 bei Prof. Dr. Uwe Kastens

## List Scheduling

**Input:** data dependence graph  
**Output:** a schedule of **at most k operations per cycle**, such that all **dependences point forward**; DDG arranged in levels

**Algorithm:** A **ready list** contains all operations that are **not yet scheduled**, but whose **predecessors are scheduled**  
 Iterate: **select** from the ready list up to k operations for the next cycle (heuristic), **update** the ready list

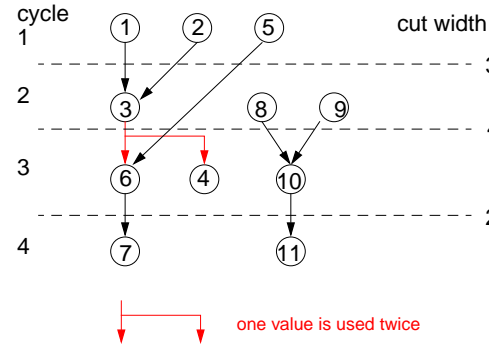


- Algorithm is **optimal** only for **trees**.
- Heuristic:** Keep ready list sorted by distance to an end node, e. g.  
 (1 2 5) (8 9 3) (6 10 4) (7 11)  
 without this heuristic:  
 (1 8 9) (2 5 10) (3 11) (6 4) (7)  
 ( ) operations in one cycle

**Critical paths** determine minimal schedule length: e. g. 1 -> 3 -> 6 -> 7

## Variants and Restrictions for List Scheduling

- Allocate **as soon as possible**, ASAP (C-5.3); as **late** as possible, ALAP
- Operations have **unit execution time** (C-5.3); **different execution times:** selection avoids conflicts with already allocated operations
- Operations only on **specific functional units** (e. g. 2 int FUs, 2 float FUs)
- Resource restrictions** between operations, e. g.  $\leq 1$  load or store per cycle



Scheduled DDG models **number of needed registers:**

- arc represents the use of an intermediate result
- cut width** through a level gives the number of **registers needed**

The tighter the schedule the more registers are needed (*register pressure*).

## Instruction Scheduling for Pipelining

**Instruction pipeline with 3 stages:**



**Dependent instructions** may not follow one another immediately.

Schedule rearranges the operation sequence, to minimize the number of delays:

**without scheduling:**

- 1: t1 := a
- 2: t2 := b
- 3: t3 := t1 + t2
- 4: x := t3
- 5: t4 := c
- 6: t5 := t3 + t4
- 7: y := t5
- 8: t6 := d
- 9: t7 := e
- 10: t8 := t6 + t7
- 11: z := t8

- 1: t1 := a
- 2: t2 := b
- 3: t3 := t1 + t2 **with scheduling**
- 4: x := t3 **no delays**
- 5: t4 := c
- 6: t5 := t3 + t4
- 7: y := t5
- 8: t6 := d
- 9: t7 := e
- 10: t8 := t6 + t7
- 11: z := t8

## Instruction Scheduling Algorithm for Pipelining

**Algorithm:** modified list scheduling:

Select from the ready list such that the selected operation

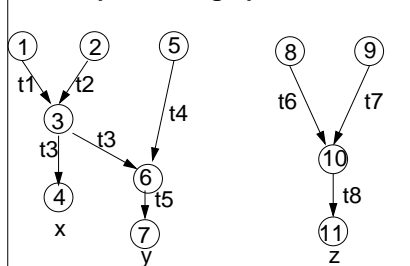
- has a sufficient **distance to all predecessors** in DDG
- has **many successors** (heuristic)
- has a **long path to the end node** (heuristic)

Insert an empty operation if none is selectable.

Ready list with additional information:

opr.	1	2	5	8	9	3	6	4	10	7	11
succ #	1	1	1	1	1	2	1	0	1	0	0
to end	3	3	2	2	2	2	1	1	1	0	0
sched. cycle	1	2	3	5	6	4	7	9	8	10	11

**data dependence graph**



cycle	1:	t1	:= a	
2:	t2	:= b		
3:	t4	:= c		
4:	t3	:= t1 + t2		<b>with scheduling</b>
5:	t6	:= d		
6:	t7	:= e		
7:	t5	:= t3 + t4		
8:	t8	:= t6 + t7		
9:	x	:= t3		
10:	y	:= t5		
11:	z	:= t8		

## Reused registers: anti- and output-dependences

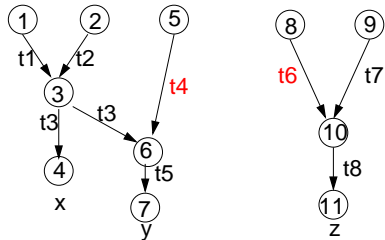
C-5.6b

$u \rightarrow v$  **flow-dependence:**  
u writes before v uses

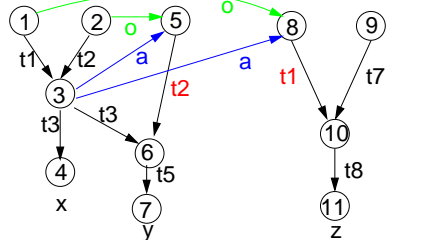
$u \xrightarrow{a} v$  **anti-dependence:**  
u uses a value before v overwrites it

$u \xrightarrow{o} v$  **output-dependence:**  
u writes before v overwrites

### DDG with symbolic registers ti flow-dependences only



### DDG with reused registers ti flow, anti-, and output-dependences



© 2011 bei Prof. Dr. Uwe Kastens

## DDG with Loop Carried Dependences

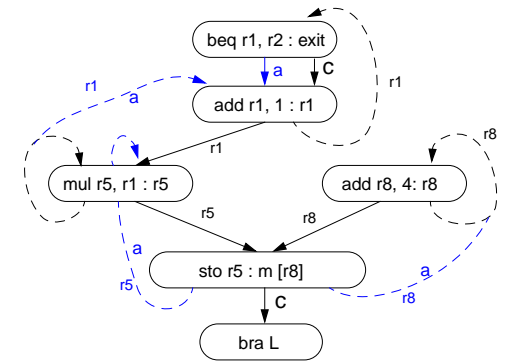
C-5.6d

Factorial computation:

**program:**  
i = 0; f = 1;  
while ( i != n )  
{ i = i + 1;  
f = f \* i;  
m[i] = f;  
}

**seq. machine code:**  
L: beq r1, r2 : exit  
add r1, 1 : r1  
mul r5, r1 : r5  
add r8, 4 : r8  
sto r5 : m[r8]  
bra L

**Data dependence graph:**



$u \rightarrow v$  **flow-dependence:**  
u writes before v uses

$u \dashrightarrow v$  **flow-dependence** into subsequent iteration

$u \xrightarrow{a} v$  **anti-dependence:**  
u uses a value before v overwrites it

$u \xrightarrow{o} v$  **output-dependence:**  
u writes before v overwrites

$u \xrightarrow{C} v$  **control-dependence:**  
u has to be executed before v (u or v may branch)

© 2011 bei Prof. Dr. Uwe Kastens

## Loop unrolling

C-5.6u

Loop unrolling: A technique for parallelization of loops.

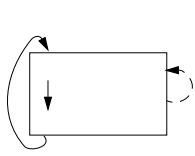
A single loop body does not exhibit enough parallelism => sparse schedule.  
**Schedule the code (copies) of several adjacent iterations together**

=> more compact schedule

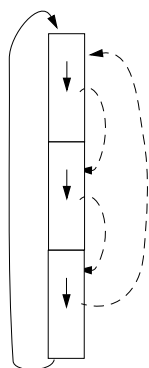
sequential loop



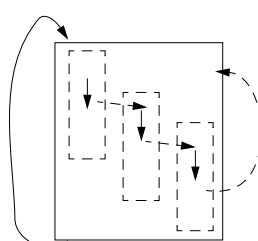
parallel schedule for single body



unrolled loop (3 times)



parallel schedule for unrolled loop



Prologue and epilogue needed to take care of iteration numbers that are not multiples of the unroll factor

© 2009 bei Prof. Dr. Uwe Kastens

## Software Pipelining

C-5.7

Software Pipelining: A technique for parallelization of loops.

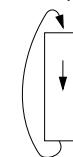
A single loop body does not exhibit enough parallelism => sparse schedule.  
**Overlap the execution of several adjacent iterations** => compact schedule

The **pipelined loop body**

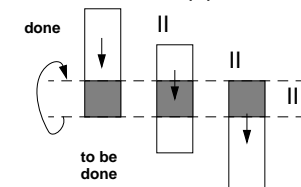
has **each operation** of the original sequential body, they belong to **several iterations**, they are **tightly scheduled**, its length is the **initiation interval II**, is **shorter** than the original body.

**Prologue, epilogue:** initiation and finalization code

sequential



software pipelined



prologue

pipelined loop

epilogue

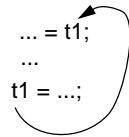
© 2011 bei Prof. Dr. Uwe Kastens



## Transform Loops by Software Pipelining

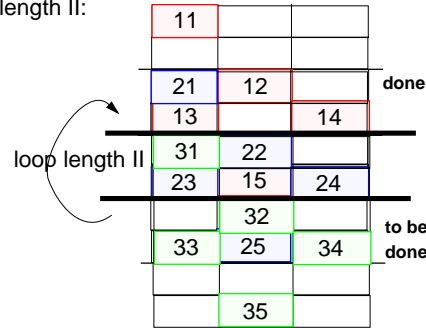
### Technique:

1. **Data dependence graph** for the loop body, include **loop carried dependences**.
2. Chose a **small initiation interval II** - not smaller than #instructions / #FUs
3. Make a „**Modulo Schedule**“ s for the loop body: Two instructions can not be scheduled on the same FU,  $i_1$  in cycle  $c_1$  and  $i_2$  in cycle  $c_2$ , if  $c_1 \bmod II = c_2 \bmod II$
4. If (3) does not succeed without conflict, increase II and repeat from 3
5. Allocate the instructions of s in the new loop of length II:  $i_j$  scheduled in cycle  $c_j$  is allocated to  $c_j \bmod II$
6. Construct prologue and epilogue.



Modulo schedule for a loop body

cycle	0	1	2	3	4	5
0	11					
1						
2		12				
3	13		14			
4						
5		15				



## Result of Software Pipelining

t	t <sub>m</sub>	ADD	MUL	MEM	CTR
0	0	L:			beq r1, r2:exit
1	1	add r1, 1: r1			
2	0	add r8, 4: r8	mul r5, r1: r5		
3	1		... mul		
4	0			sto r5: m r8	
5	1			... sto	
6	0				
7	1				bra L

4 dedicated FUs schedule of the loop body for II = 2  
mul and sto need 2 cycles  
add and sto in t<sub>m</sub>=0, sto reads r8 before add writes it  
bra not in cycle 6, it collides with beq; t<sub>m</sub>=0

t	t <sub>m</sub>	ADD	MUL	MEM	CTR	
0	0				beq r1:r2:exit	
1	1	add r1, 1: r1				
2	0	add r8, 4: r8	mul r5, r1: r5		beq r1: r2: ex	
3	1	add r1, 1: r1	... mul			
4	0	L:	add r8, 4: r8	mul r5, r1: r5	sto r5: m r8	beq r1: r2: ex
5	1		add r1, 1: r1	... mul	... sto	bra L
6	1	ex:	... mul	... sto		
7	0			sto r5: m r8		
8	1			... sto		
9	0				bra exit	

prologue  
software pipeline with II = 2  
epilogue

## 5.2 / 6. Data Parallelism: Loop Parallelization

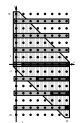
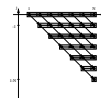
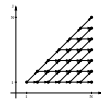
**Regular loops** on orthogonal data structures - parallelized for **data parallel** processors

Development steps (automated by compilers):

- **nested loops** operating on **arrays**, sequential execution of iteration space
- analyze **data dependences**  
data-flow: definition and use of array elements
- **transform loops**  
keep data dependences forward in time
- **parallelize inner loop(s)**  
map to field or vector of processors
- **map arrays to processors**  
such that many accesses are local, transform index spaces

```

DECLARE B[0..N,0..N+1]
FOR I := 1 .. N
  FOR J := 1 .. I
    B[I,J] :=
      B[I-1,J]+B[I-1,J-1]
  END FOR
END FOR
    
```



## Iteration space of loop nests

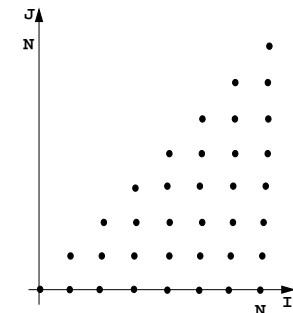
**Iteration space** of a loop nest of depth n:

- **n-dimensional space of integral points** (polytope)
- each point  $(i_1, \dots, i_n)$  represents an execution of the innermost loop body
- loop bounds are in general not known before run-time
- iteration need not have orthogonal borders
- iteration is elaborated sequentially

example:  
computation of Pascal's triangle

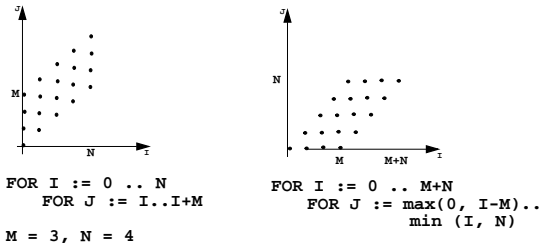
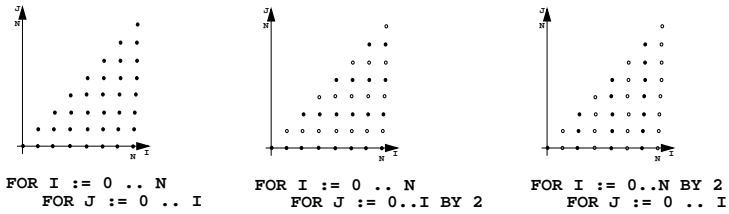
```

DECLARE B[-1..N,-1..N]
FOR I := 0 .. N
  FOR J := 0 .. I
    B[I,J] :=
      B[I-1,J]+B[I-1,J-1]
  END FOR
END FOR
    
```





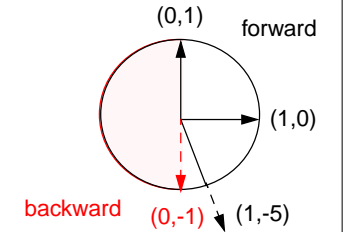
### Examples for Iteration spaces of loop nests



### Data Dependences in Iteration Spaces

#### Data dependence from iteration point i1 to i2:

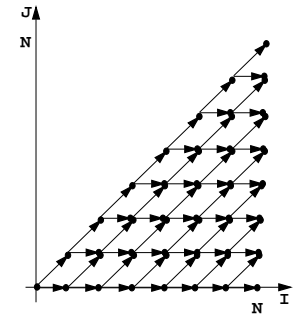
- Iteration i1 computes a value that is used in iteration i2 (flow dependence)
- relative **dependence vector**  $d = i2 - i1 = (i2_1 - i1_1, \dots, i2_n - i1_n)$  holds for all iteration points except at the border
- Flow-dependences can **not be directed against the execution order**, can not point backward in time: each dependence vector must be **lexicographically positive**, i. e.  $d = (0, \dots, 0, d_i, \dots), d_i > 0$



Example:  
Computation of Pascal's triangle

```

DECLARE B[-1..N,-1..N]
FOR I := 0 .. N
  FOR J := 0 .. I
    B[I,J] :=
      B[I-1,J]+B[I-1,J-1]
  END FOR
END FOR
    
```



### Loop Transformation

The **iteration space** of a loop nest is transformed to **new coordinates**. Goals:

- execute innermost loop(s) in parallel**
- improve **locality** of data accesses; **in space**: use storage of executing processor, **in time**: reuse values stored in cache
- systolic** computation and communication scheme

Data dependences must **point forward in time**, i.e. **lexicographically positive** and **not within parallel dimensions**

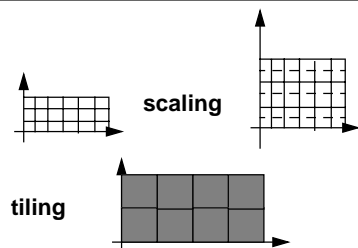
#### linear basic transformations:

- Skewing**: add iteration count of an outer loop to that of an inner one
- Reversal**: flip execution order for one dimension
- Permutation**: exchange two loops of the loop nest

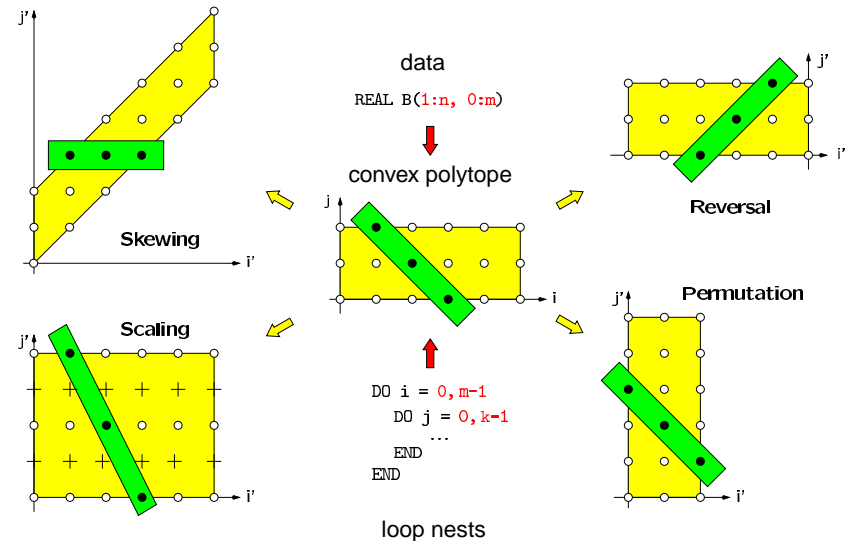
#### SRP transformations (next slides)

#### non-linear transformations, e. g.

- Scaling**: stretch the iteration space in one dimension, causes gaps
- Tiling**: introduce **additional inner loops** that **cover tiles** of fixed size



### Transformations of





### Use of Transformation Matrices

- Transformation matrix **T** defines **new iteration counts** in terms of the old ones:  $T * i = i'$

e. g. Reversal 
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} i \\ -j \end{pmatrix} = \begin{pmatrix} i' \\ j' \end{pmatrix}$$

- Transformation matrix **T** transforms old **dependence vectors** into new ones:  $T * d = d'$

e. g. 
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} * \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

- inverse Transformation matrix  $T^{-1}$  defines **old iteration counts** in terms of new ones, for transformation of index expressions in the loop body:  $T^{-1} * i' = i$

e. g. 
$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} * \begin{pmatrix} i' \\ j' \end{pmatrix} = \begin{pmatrix} i' \\ -j' \end{pmatrix} = \begin{pmatrix} i \\ j \end{pmatrix}$$

- concatenation of transformations** first  $T_1$  then  $T_2$ :  $T_2 * T_1 = T$

e. g. 
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

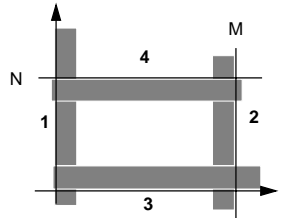
### Inequalities Describe Loop Bounds

The bounds of a loop nest are described by a **set of linear inequalities**. Each **inequality separates the space** in „inside and outside of the iteration space“:

$$B * i \leq c$$

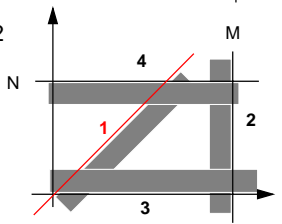
example 1 
$$\begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} \leq \begin{pmatrix} 0 \\ M \\ 0 \\ N \end{pmatrix}$$

- 1  $-i \leq 0$
- 2  $i \leq M$
- 3  $-j \leq 0$
- 4  $j \leq N$



example 2 
$$\begin{pmatrix} -1 & 1 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} i \\ j \end{pmatrix} \leq \begin{pmatrix} 0 \\ M \\ 0 \\ N \end{pmatrix}$$

- 1  $-i + j \leq 0$
  - 2  $i \leq M$
  - 3  $-j \leq 0$
  - 4  $j \leq N$
- transformed



**positive** factors represent **upper** bounds  
**negative** factors represent **lower** bounds

- 1, 4:  $j \leq \min(i, N)$
- 1+3:  $0 \leq i$
- 2:  $i \leq M$

### Transformation of Loop Bounds

The inverse of a transformation matrix  $T^{-1}$  transforms a set of inequalities:  $B * T^{-1} i' \leq c$

skewing 
$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$
 inverse 
$$\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

$$B * T^{-1} = \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ -1 & 1 \end{pmatrix}$$

example 1  
new bounds:

$$B * T^{-1} * \begin{pmatrix} i' \\ j' \end{pmatrix} \leq \begin{pmatrix} 0 \\ M \\ 0 \\ N \end{pmatrix}$$

- 1  $-i' \leq 0$
- 2  $i' \leq M$
- 3  $i' - j' \leq 0$
- 4  $-i' + j' \leq N$

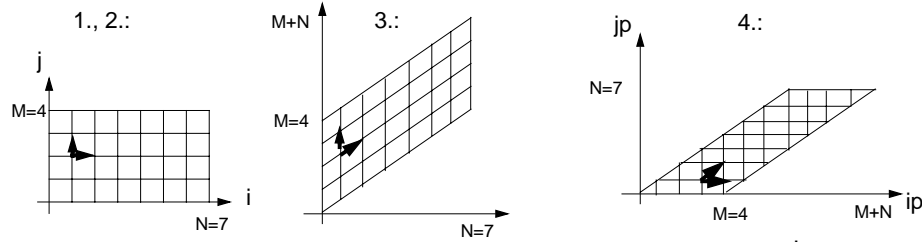
### Example for Transformation and Parallelization of a Loop

```
for i = 0 to N
  for j = 0 to M
    a[i, j] = (a[i, j-1] + a[i-1, j]) / 2;
```

Parallelize the above loop.

- Draw the iteration space.
- Compute the dependence vectors and draw examples of them into the iteration space. Why can the inner loop not be executed in parallel?
- Apply a skewing transformation and draw the iteration space.
- Apply a permutation transformation and draw the iteration space. Explain why the inner loop now can be executed in parallel.
- Compute the matrix of the composed transformation and use it to transform the dependence vectors.
- Compute the inverse of the transformation matrix and use it to transform the index expressions.
- Specify the loop bounds by inequalities and transform them by the inverse of the transformation matrix.
- Write the complete loops with new loop variables ip and jp and new loop bounds.

### Solution of the Transformation and Parallelization Example

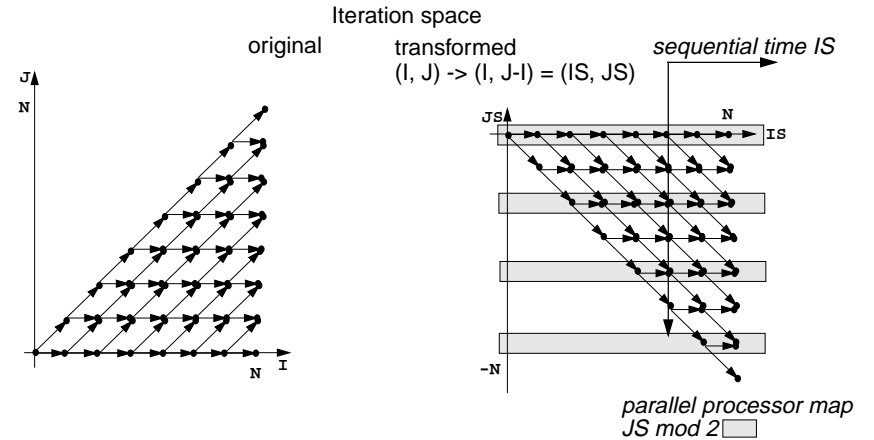


5.:  $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$       $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$      6.: Inverse  $\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$

7. Bounds:  $B$       $c$       $B * T^{-1}$   
 orig.:  $\begin{pmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{pmatrix}$       $\begin{pmatrix} 0 \\ N \\ 0 \\ M \end{pmatrix}$      new:  $\begin{pmatrix} 0 & -1 \\ 0 & 1 \\ -1 & 1 \\ 1 & -1 \end{pmatrix}$      1  $-jp \leq 0$      1, 3  $\Rightarrow 0 \leq ip$   
 2  $jp \leq N$      2, 4  $\Rightarrow ip \leq M+N$   
 3  $-ip+jp \leq 0$      1, 4  $\Rightarrow \max(0, ip-M) \leq jp$   
 4  $ip - jp \leq M$      2, 3  $\Rightarrow jp \leq \min(ip, N)$

```
8. for ip = 0 to M+N
    for jp = max(0, ip-M) to min(ip, N)
        a[jp, ip-jp] = (a[jp, ip-jp-1] + a[jp-1, ip-jp]) / 2;
```

### Transformation and Parallelization



```
DECLARE B[-1..N,-1..N]
FOR I := 0 .. N
  FOR J := 0 .. I
    B[I,J] :=
      B[I-1,J]+B[I-1,J-1]
  END FOR
END FOR
```

```
DECLARE B[-1..N,-1..N]
FOR IS := 0 .. N
  FOR JS := -IS .. 0
    B[IS,JS+IS] :=
      B[IS-1,JS+IS]+B[IS-1,JS-1+IS]
  END FOR
END FOR
```

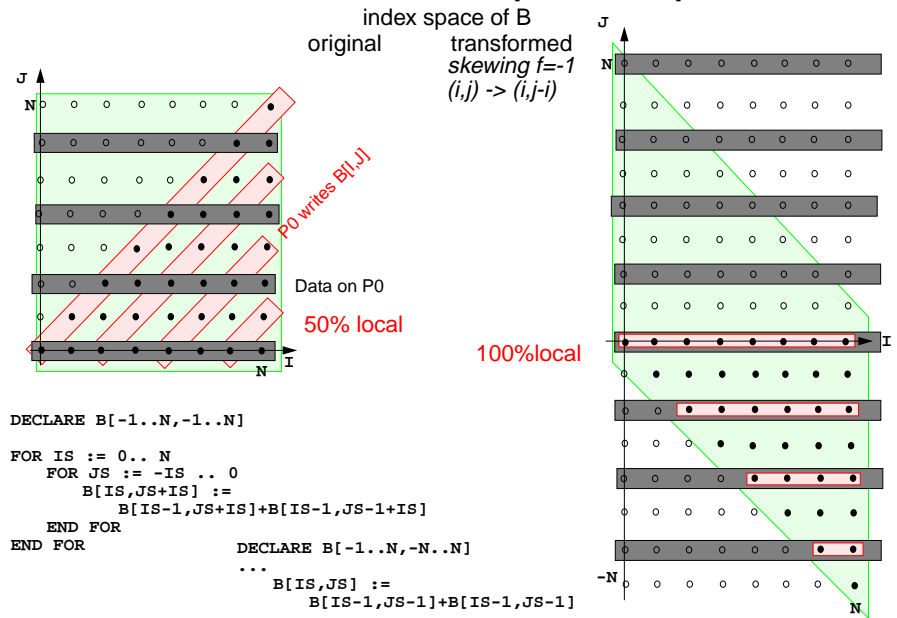
### Data Mapping

**Goal:**  
 Distribute array elements over processors, such that as many accesses as possible are local.

**Index space** of an array:  
 n-dimensional space of integral index points (polytope)

- same properties as iteration space
- same mathematical model
- same transformations are applicable (Skewing, Reversal, Permutation, ...)
- no restrictions by data dependences

### Data distribution for parallel loops



```
DECLARE B[-1..N,-1..N]
FOR IS := 0 .. N
  FOR JS := -IS .. 0
    B[IS,JS+IS] :=
      B[IS-1,JS+IS]+B[IS-1,JS-1+IS]
  END FOR
END FOR
...
DECLARE B[-1..N,-N..N]
...
B[IS,JS] :=
  B[IS-1,JS-1]+B[IS-1,JS-1]
```

## Check Your Knowledge (1)

C-6.1

### Optimization, CFA:

1. Explain graphs that are used in program analysis.
2. Which optimizing transformations need analysis of execution paths?
3. Which optimizing transformations do not need analysis of execution paths?
4. Give an example for a pair of transformations such that one enables the other.
5. Define the control-flow graph. Describe transformations on the CFG.
6. Define the dominator relation. What is it used for?
7. Describe an algorithm for computing dominator sets.
8. Define natural loops.
9. What is the role of the loop header and of the pre-header.
10. Show a graph that has a cycle but no natural loop.
11. Define induction variables, and explain the transformation technique.

© 2012 bei Prof. Dr. Uwe Kastens

## Check Your Knowledge (2)

C-6.2

### Optimization, DFA:

12. Describe the schema for DFA equations for the four problem categories.
13. Explain the relation of the meet operator, the paths in the graph, and the DFA solutions.
14. Describe the DFA problem reaching definitions.
15. Describe the DFA problem live variables.
16. Describe the DFA problem available expressions.
17. Describe the DFA problem copy propagation.
18. Describe the DFA problem constant propagation.
19. Describe the iterative DFA algorithm; its termination; its complexity.
20. Describe an heuristic improvement of the iterative DFA algorithm.
21. Extend constant propagation to interval propagation for bounds checks. Explain the interval lattice.
22. What is the role of lattices in DFA?
23. Describe lattices that are common for DFA.

© 2011 bei Prof. Dr. Uwe Kastens

## Check Your Knowledge (3)

C-6.3

### Object Oriented Program Analysis:

24. Describe techniques to reduce the number of arcs in call graphs.
25. Describe call graphs for object oriented programs.
26. Describe techniques to reduce the number of arcs in object oriented call graphs.

### Code Generation, Storage mapping:

27. Explain the notions of storage classes, relative addresses, alignment, overlay.
28. Compare storage mapping of arrays by pointer trees to mapping on contiguous storage.
29. Explain storage mapping of arrays for C. What is different for C, for Fortran?
30. For what purpose are array descriptors needed? What do they contain?
31. What is the closure of a function? In which situation is it needed?
32. Why must a functional parameter in Pascal be represented by a pair of pointers?
33. What does an activation record contain?
34. Explain static links in the run-time stack. What is the not-most-recent property?
35. How do C, Pascal, and Modula-2 ensure that the run-time stack discipline is obeyed?
36. Why do threads need a separate run-time stack each?

© 2011 bei Prof. Dr. Uwe Kastens

## Check Your Knowledge (4)

C-6.4

37. Explain the code for function calls in relation to the structure of activation records.
38. Explain addressing relative to activation records.
39. Explain sequences for loops.
40. Explain the translation of short circuit evaluation of boolean expressions. Which attributes are used?
41. Explain code selection by covering trees with translation patterns.
42. Explain a technique for tree pattern selection using 3 passes.
43. Explain code selection using parsing. What is the role of the grammar?

### Register Allocation

44. How is register windowing used for implementation of function calls?
45. Which allocation technique is applied for which program context?
46. Explain register allocation for expression trees. Which attributes are used?
47. How is spill code minimized for expression trees?
48. Explain register allocation for basic blocks? Relate the spill criteria to paging techniques.
49. Explain register allocation by graph coloring. What does the interference graph represent?
50. Explain why DFA life-time analysis is needed for register allocation by graph coloring.

© 2002 bei Prof. Dr. Uwe Kastens

## Check Your Knowledge (5)

### Instruction Scheduling

51. What does instruction scheduling mean for VLIW, pipeline, and vector processors?
52. Explain the kinds of arcs of DDGs (flow, anti, output).
53. What are loop carried dependences?
54. Explain list scheduling for parallel FUs. How is the register need modelled?  
Compare it to Belady's register allocation technique.
55. How is list scheduling applied for arranging instructions for pipeline processors?
56. Explain the basic idea of software pipelining. What does the initiation interval mean?

### Loop Parallelization

57. Explain dependence vectors in an iteration space.  
What are the admissible directions for sequential and for parallelized innermost loops?
58. What is tiling, what is scaling?
59. Explain SRP transformations.
60. How are the transformation matrices used?
61. How are loop bounds transformed?
62. Parallelize the inner loop of a nest that has dependence vectors  $(1,0)$  and  $(0, 1)$ ?